RANK SENSITIZATION OF COUNTRIES IN HUMAN DEVELOPMENT REPORTS OF UNDP IN VIEW OF KAKWANI'S APPROACH FOR ACHIEVEMENT INDICATOR

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Abstract

In this paper, we study variations and sensitization in the ranks of various countries covered in selective Human Development Reports of UNDP from the stance of Kakwani's (1993) approach of Achievement Indicator when built into different scenarios of formulating utility function of income component of Human Development Index.

Keywords: Human Development Index, Income-transforming Utility Function, Achievement Indicator Approach

1. Introduction

The first Human Development Report HDR(1990) brought out by United Nations Development Programme (UNDP) introduced a new way of measuring development by combining indicators of life expectancy, educational attainment and income into a composite Human Development Index (HDI) that purports to assess the scenario of average achievement in a country in three basic dimensions of Human Development, viz. (a) a long and healthy life, as measured by life expectancy at birth (b) knowledge, as measured by a synthetic combination of adult literacy rate and the combined gross enrollment ratio for primary, secondary and tertiary schools and (c) a decent standard of living, as measured by GDP per capita in purchasing power parity US dollars. The HDI combines normalized measures of life expectancy, literacy, educational attainment and GDP per capita for countries worldwide and a major breakthrough came in terms of creation of a single statistic, which would serve as a frame of reference for both social and economic development.

The HDI assumes vast significance in today's world embracing wide range of disciplines. One may easily measure and compare spatial and time-series progress in human development amongst different countries. The HDI commands adequate popularity due to its transparency, being simple to construct and easy to comprehend. Nevertheless, it remained under variety of conceptual and rudimentary criticisms especially during first decade of its existence. Some of the salient criticisms pertained to (a) non-inclusion of several other dimensions of human well-being apart from longevity, education and decent standard of living, their weighting mechanism in the HDI and certain fundamental flaws in conceiving Educational Attainment Indicator as well as the Indicator for Income component. One may refer to Allen C. Kelley (1991), Mark McGillavray and Howard White (1993), Meghnad Desai (1991), Michael Hopkins (1991), Harald Trabold-Nubler (1991) for relevant details. Bhatnagar (2001a)

has reviewed the methodology adopted in the HDRs and studied the impact with reference to India.

The maximum and minimum values of the components involved in the computation of the HDI from inception till 1993, used to be the extreme performance levels actually observed for different countries. However, with effect from 1994 only fixed normative values were used as the maximum and minimum goalposts in all HDRs brought out by UNDP. The HDI sets a minimum and a maximum goalpost for each constituent in order to ascertain where each country stands with reference to any particular constituent of the HDI when observed in relation to the prescribed goalposts. The component of educational attainment in the HDI comprises of (a) adult literacy rate and (b) combined gross enrolment ratio for primary, secondary and tertiary schooling with minimum goalpost for each category as 0% and the maximum as 100%. The component of life expectancy in the HDI is calculated using a minimum value for life expectancy at birth as 25 years and maximum value thereof as 85 years. For the income component, the minimum goalpost adopted by the HDRs is $100 (PPP) while the maximum level stands at $40,000 (PPP). The HDI uses the logarithm of income, to reflect the diminishing importance of income with increasing GDP.

The first ever HDR of 1990 deployed a truncated logarithmic utility function, giving zero weights to the income levels above a particular cut-off level, implying thereby that any additional income levels beyond the cut-off level won't yield any further utility. During 1991 to 1998 UNDP in its HDR prescribed a multi-step utility function with Atkinson's formulation as radix, used for computing the extent of utility derived from any given level of real GDP per capita. According to the Atkinson based multi-step utility function adopted in the HDR during 1991 to 1998, the effect of diminishing marginal utility was introduced beyond a certain level of income termed as threshold income level. However, for real GDP per capita of any country below the threshold income level, no discounting or adjustment was envisaged. The range beyond the threshold income level sub-divided into income range segments of equal width with span of each income range segment being equal to the numerical value of the threshold income level, used the Atkinson based multi-step utility function differently for different income range segments beyond the threshold income level for discounting the values of utility or welfare corresponding to the income range segment. Formulation of multi-step utility function in HDR during 1991 to 1998 has been found to suffer from unsatisfactory mathematical treatment meted out to Atkinson's function. During analytical review with special reference to the income component, the inherent weaknesses in the formulation of the Atkinson based multi-step utility function have been exposed by Bhatnagar (2001-b).

From 1999, UNDP in all its subsequent HDRs discarded the Atkinson based multi-step utility function and switched back to the approach akin to that of HDR 1990 for formulation of the utility function- the only difference being that from HDR 1999 onwards a non-truncated logarithmic function of the real GDP per capita has replaced the truncated formulation of HDR 1990. Going much more beyond UNDP's justification for digressing from its own earlier approach involving threshold-income's concept, the fundamental issues behind shifting from the multistep Atkinson based approach for framing utility function have been duly investigated in depth by Bhatnagar (2002).

Chaubey (1998) provided a condition under which UNDP formulation would fail on the yardstick of principle of diminishing marginal utility while considering a simplified situation of two-fold decomposition of the entire Income-range as (a) below the threshold level and (b) above the threshold level. While generalizing Chaubey (1998)'s result under multi-step fragmentation of the range of income, Bhatnagar (2001-b) has explicitly identified the precise income bands within each income interval, corresponding to which the Atkinson based formulation is bound to fail on the yardstick of the principle of diminishing marginal utility. A viable replacement of the Atkinson based multi-step utility function has also been proposed by Bhatnagar i.b.i.d by considering the fragmentation of the income range beyond threshold income level in the multiples of twice the numerical value of the threshold income level and the same was shown by him to be free from the problems of 'incompatibility of units and dimension' and did not also flout the 'principle of diminishing marginal utility'.

Bhatnagar (2002) studied the effect on the ranks of the countries by considering the approach of UNDP vis-a-vis his alternative formulation, besides observing the variations in ranks due to Chaubey's two-step utility function for the same period. Evidently, it is not only the modification in the utility function which brings about changes in the relative ranking of the countries but the varying threshold income level alone also may bring about variations in rankings. The effects due to change in the utility function from Atkinson based formulation to the alternative multi-step replacement simultaneously along with the variations in the threshold income levels have also been captured by Bhatnagar i.b.i.d.

The period from 1995 to 1998 in the era of Human Development Reports has been observed to be of typical significance considering stability in methodology with regard to the formulation of multi-step utility function. In this paper we study the variations and sensitization in the ranks of the countries of HDR for this particular period from 1995 to 1998, while adopting Kakwani's Achievement Indicator approach.

2. Kakwani's Achievement Indicator Approach

A variant to the approach already available in literature can be considered by adopting Kakwani (1993)'s concept of an Achievement Indicator defined by the function

$$F(y,M,m)=1-(g(M-y))/(g(M-m)) \qquad (1)$$

where $g(.)$ is an increasing function with limiting value of $g(y)$ as zero when y tends to zero. Let us consider the well-known utility function due to Atkinson defined as W $(y)=y^{(1-\varepsilon)}/(1-\varepsilon)$, $0\leq\varepsilon<1$. Since its first derivative with respect to 'y' is greater than zero and the limit of $W(y)$ as 'y' tends to zero is also zero, the Atkinson's function can be used as function $g(..)$ in Equation (1). The variable $(M-y)$ is taken as the argument in the Atkinson's function. From equation (1) we have

$$F(y,M,m)=1-[\{1/(1-\varepsilon)\}(M-y)^{(1-\varepsilon)}]/[\{1/(1-\varepsilon)\}(M-m)^{(1-\varepsilon)}]$$

Or that

$$F(y,M,m)=1-\{(M-y)/(M-m)\}^{(1-\varepsilon)} \qquad (2)$$

Let X_1, X_2, X_3 and X_4 represent the values for life expectancy at birth, adult literacy rate, gross enrolment ratios and real GDP per capita for a country in the HDR data. Any country's deprivation in each of these components is considered as the ratio of the extent of deviation of the actual value from the maximum goalpost for that component

to the total difference or range between the maximum goalpost and the minimum goalpost. If M and m denote respectively the maximum and minimum values for the goal posts and 'y' the actual value for a particular component, the corresponding deprivation (say, d) expressed as d= a(M-y), where 'a' is a constant equal to reciprocal of difference between M and m, is a linear function in the difference between the maximum and actual value.

Following Equation (2), we introduce the values of the Kakwani's Achievement Indicator for life expectancy at birth, adult literacy rate, combined gross enrolment ratios and real GDP per capita and compute the modified values of HDI with a view to study the sensitization in the ranking of various countries. The scope of study caters to the broad categories/ scenario as discussed in subsequent Sections.

3. Impact of Kakwani's Achievement Indicator Approach over Traditional Approach of UNDP Using Multi-step Utility Function

Out of the entire set of plausible values for parameter '$\in$' between zero and unity, we have considered three sets of illustrative values for '$\in$' as 1/2 , 1/3 and 2/3 for the computation purposes. Since our objective involves only the assessment of relative ranking of different countries, taking any different value for the parameter '$\in$' would not be of material significance.

Taking '$\in$' as 1/2, 1/3 and 2/3, the life expectancy (at birth) Achievement Indicator due to Kakwani's approach is computed by following Equation

$$L_A = 1 - \{(85-X_1)/(85-25)\}^{(1-\in)} \tag{3}$$

Following the similar weighting method as adopted by UNDP in respect of its Educational Attainment index, the adult literacy rate and combined gross enrolment ratios are combined together for each country with $2/3^{rd}$ and $1/3^{rd}$ weights under Kakwani's Approach using following Equation:

$$E_A = (2/3)*(1-\{(100-X_2)/100\}^{(1-\in)})+(1/3)*(1-\{(100-X_3)/100\}^{(1-\in)}) \tag{4}$$

The values of Threshold Income Level (abbreviated as 'TIL') in the HDRs from 1995 to 1998 were 5120 (PPP\$), 5711(PPP\$), 5835(PPP\$) and 5990(PPP\$) respectively. The corresponding adjusted maximum GDP per capita under HDRs were taken as 5448 (PPP\$), 6040 (PPP\$), 6154 (PPP\$) and 6311 (PPP\$) respectively by discounting the maximum value of 40,000 (PPP\$) with the help of the Atkinson based multi-step utility function. If real GDP per capita is less than or equal to 'TIL' then the same is used as the adjusted real GDP per capita, otherwise the adjusted real GDP per capita is computed using Atkinson based multi-step utility function. The GDP Achievement indicator deploying Kakwani's approach is computed by using the Equation

$$I_A = [1-\{(\text{adj. Max. GDP per capita-adj. Real GDP per capita})/ (\text{adj. Max. GDP per capita-100})\}^{(1-\in)}] \tag{5}$$

The value for variant formulation of UNDP's HDI taking into consideration the Kakwani's Achievement Indicator Approach as the base is obtained as the average of L_A, E_A and I_A for the country. Fresh ranks are computed for all the countries included

in the UNDP's Human Development Reports. The variations in their ranks [upward and downward depicted as 'up' and 'dn' respectively] when compared with the corresponding ranks given in HDR from 1995 to 1998 (without Kakwani's Achievement Indicator approach) have been assessed for the chosen illustrative values of '$\in$'. The countries which have been affected the most in the process are indicated as under:

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Oman	11up	16up	9up	Malta	9dn	12dn	5dn
Slovakia	10up	11up	3up	Costa Rica	9dn	15dn	5dn
Georgia	9up	14up	5up	Portugal	9dn	11dn	7dn
Czech Rep.	9up	10up	4up	Albania	9dn	11dn	6dn
Armenia	8up	11up	5up	Dom. Rep.	8dn	8dn	7dn
Hungary	8up	13up	6up	Jamaica	7dn	8dn	5dn

Table-1: Most affected countries due to achievement approach in HDR 1995

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Oman	16up	18up	9up	Jordan	12dn	15dn	9dn
Armenia	15up	22up	8up	Dom. Rep.	9dn	13dn	5dn
Vietnam	9up	13up	4up	Turkey	9dn	13dn	4dn
Grenada	9up	11up	5up	Syrian Arab Rep.	9dn	10dn	5dn
Czech. Rep.	8up	8up	3up	Tunisia	8dn	11dn	5dn
Hungary	7up	9up	5up	Belize	8dn	13dn	3dn

Table-2: Most affected countries due to achievement approach in HDR 1996

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Oman	9up	14up	7up	Dom. Rep.	9dn	11dn	6dn
Armenia	9up	15up	6up	Spain	8dn	10dn	5dn
Vietnam	8up	9up	6up	Indonesia	7dn	9dn	5dn
Czech. Rep.	8up	9up	7up	Mongolia	7dn	8dn	6dn
Saudi Arabia	7up	7up	5up	South Africa	7dn	10dn	4dn
Kuwait	6up	7up	4up	Jordan	6dn	9dn	4dn

Table-3: Most affected countries due to achievement approach in HDR 1997

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Armenia	10up	16up	4up	Indonesia	11dn	12dn	8dn
Uzbekistan	9up	15up	5up	South Africa	9dn	13dn	5dn
Georgia	9up	20up	5up	Spain	8dn	10dn	5dn
Czech. Rep.	8up	8up	5up	Maldives	7dn	8dn	5dn
Vietnam	8up	9up	4up	Mongolia	7dn	9dn	4dn
Tazakistan	8up	11up	6up	Costa Rica	7dn	8dn	6dn

Table-4: Most affected countries due to achievement approach in HDR 1998

4. Impact of Kakwani's Achievement Indicator Approach Using Chaubey's Two-Step Utility Function Against Traditional Approach of UNDP Using Multi-Step Utility Function

Under a Two- step formulation of utility function envisaging the entire income-range as divided into two segments namely, below the threshold income level and above threshold income level, Chaubey (1998) proposed for all real GDP per capita income levels $\geq$ TIL,

$$\text{GDP}_{chaubey} = \text{TIL} + \text{TIL} * \log (\text{real GDP per capita/ TIL}) \tag{6}$$

and for real GDP per capita income levels $\leq$ TIL

$$\text{GDP}_{chaubey} = \text{TIL} \tag{7}$$

In this Section, we compute the fresh values for HDI by considering Kakwani's Achievement Indicator approach while replacing the Traditional Atkinson-based Multi-step utility function with Chaubey (1998)'s Two-step utility function. For the life expectancy at birth and Educational attainment, the Achievement Indicator due to Kakwani's approach can be computed as usual by Equation (3) and (4). For working out the GDP Achievement Indicator, the real GDP per capita income levels are adjusted using Chaubey's formulation specified as per Equations (6) and (7).

The values of 'M' and 'm' corresponding to maximum and minimum goalposts thus worked out are indicated below:

Year	Threshold income (in PPP$)	M (in PPP$)	M (in PPP$)
1998	5990	100	10930
1997	5835	100	10713
1996	5711	100	10539
1995	5120	100	9691

Table-5: Maximum and Minimum values for income component following Chaubey's formulation

Using Kakwani's Achievement Approach with Chaubey(1998)'s Two-step formulation, for different values of '$\in$' the GDP Achievement Index is computed as under:

$$I_{A(chaubey)} = [1 - \{(\text{Adusted Maximum }.\text{GDP}_{chaubey} - \text{Adjusted actual GDP}_{chaubey}) / (\text{Adjusted Maximum GDP}_{chaubey} - 100)\}^{(1-\in)}] \tag{8}$$

Corresponding adjusted Max. GDP$_{chaubey}$ in respect of the HDRs from 1995 to 1998 have been given in the Table-6. Using Equation (8), the value of achievement index corresponding to the income component for 1998, for instance would be

$$I_{A(chaubey)}=1-\{(10930- \text{Adjusted actual GDP}_{chaubey})/10830\}^{(1-\epsilon)} \qquad (9)$$

Revised values for HDI following Chaubey(1998)'s Two-step utility function formulation and using Kakwani's Achievement Indicator approach for any year would be worked out as the average of L$_A$, E$_A$ and I$_{A(chaubey)}$ for it.

Fresh rankings for different countries covered in the UNDP's HDRs under the premises of Chaubey(1998)'s Two-step formulation by considering Kakwani's Approach in lieu of the Traditional Atkinson-based Multi-step formulation have been obtained by us. The existing ranks as already recorded in the HDRs obtained by applying Atkinson based formulation of multi-step utility function (without Kakwani's approach) are compared as against the ranks obtained by considering Chaubey(1998)'s Two-step utility function, utilizing Kakwani's Achievement Indicator Approach for the period from 1995 to 1998. The countries whose ranks have been affected significantly are found as under. The variations in the ranks have been worked out for three illustrative values for '∈' as before:

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	ϵ =1/2	ϵ =2/3	ϵ =1/3		ϵ =1/2	ϵ =2/3	ϵ =1/3
Georgia	31up	39up	22up	Iran	28dn	28dn	27dn
Armenia	28up	36up	21up	Botswana	26dn	26dn	25dn
Qatar	24up	19up	25up	Tunisia	26dn	26dn	25dn
Azzerbaijan	21up	22up	16up	Turkey	26dn	29dn	20dn
Samoa (western)	20up	23up	11up	Brazil	20dn	27dn	17dn
Uzbekistan	20up	23up	16up	Algeria	18dn	18dn	18dn

Table-6: Most affected countries on replacing Traditional UNDP multi-step formulation by Chaubey's formulation under achievement approach in HDR 1995

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	ϵ =1/2	ϵ =2/3	ϵ =1/3		ϵ =1/2	ϵ =2/3	ϵ =1/3
Armenia	33up	40up	27up	Algeria	32dn	33dn	31dn
Georgia	21up	23up	15up	Iran	31dn	33dn	30dn
Krygyztan	20up	23up	15up	Botswana	29dn	30dn	28dn
Uzbekistan	19up	24up	16up	Tunisia	24dn	25dn	23dn
Grenada	19up	23up	16up	Brazil	23dn	29dn	17dn
Ajjerbaizan	18up	23up	15up	Jordan	22dn	16dn	18dn

Table-7: Most affected countries on replacing Traditional UNDP multi-step formulation by Chaubey's formulation under achievement approach in HDR 1996

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Armenia	27up	37up	21up	Iran	28dn	30dn	23dn
Luxembourg	24up	24up	24up	Algeria	24dn	26dn	23dn
Cuba	20up	22up	15up	Syrian Arab Rep.	24dn	25dn	21dn
Ukraine	20up	28up	17up	Tunisia	23dn	25dn	21dn
Moldavia Rep.	20up	30up	12up	Turkey	21dn	22dn	13dn
Kuwait	19up	16up	20up	Brazil	21dn	26dn	16dn

Table-8: Most affected countries on replacing Traditional UNDP multi-step formulation by Chaubey's formulation under achievement approach in HDR 1997

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Georgia	32up	40up	22up	Algeria	25dn	26dn	23dn
Armenia	26up	35up	21up	Iran	24dn	25dn	20dn
Uzbekistan	23up	30up	13up	Syrian Arab Rep.	23dn	24dn	21dn
Luxembourg	23up	23up	22up	Turkey	23dn	27dn	18dn
Kuwait	22up	21up	22up	Tunisia	22dn	23dn	21dn

Table-9: Most affected countries on replacing Traditional UNDP multi-step formulation by Chaubey's formulation under achievement approach in HDR 1998

5. Impact of Chaubey's Two-Step Formulation over Atkinson Based Multi-Step Utility Function Within Framework of Kakwani's Approach

When Kakwani's Achievement Indicator approach is utilized but the ranks are obtained by considering the Atkinson based multi-step utility function on one hand and Chaubey(1998)'s Two-step function on the other hand, the emerging variations in the ranks of different countries after fresh computation, reveal the most affected countries as follows:

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Georgia	22up	25up	17up	Turkey	27dn	29dn	21dn
Qatar	21up	14up	22up	Iran	26dn	25dn	25dn
Armenia	20up	25up	16up	Tunisia	25dn	25dn	24dn
Tazakistan	17up	18up	10up	Botswana	23dn	23dn	22dn
UAE	17up	15up	18up	Brazil	20dn	27dn	17dn
Samoa (western)	16up	16up	10up	Saudi Arab Rep.	20dn	24dn	9dn

Table-10: Most affected countries when under achievement approach in HDR 1995 (Atkinson based formulation is replaced with Chaubey's formulation)

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Georgia	18up	16up	18up	Algeria	31dn	29dn	32dn
Armenia	18up	18up	18up	Oman	30dn	33dn	24dn
Tazakistan	15up	18up	15up	Libyan Arab Jamah	27dn	31dn	19dn
Qatar	15up	9up	15up	Iran	25dn	24dn	29dn
Samoa (western)	14up	10up	14up	Botswana	21dn	18dn	21dn
Krygyztan	13up	12up	13up	Brazil	20dn	26dn	16dn

Table-11: Most affected countries when under achievement approach in HDR 1996 (Atkinson based formulation is replaced with Chaubey's formulation)

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Armenia	18up	22up	15up	Iran	28dn	31dn	23dn
Luxembourg	18up	13up	19up	Oman	24dn	31dn	22dn
Cuba	17up	18up	13up	Saudi Arabia	22dn	29dn	13dn
Georgia	16up	19up	11up	Libyan Arab Jamah	21dn	29dn	16dn
Moldavia Rep.	15up	17up	10up	Syrian Arab Rep.	20dn	18dn	20dn
Ajjerbaizan	14up	19up	9up	Algeria	20dn	19dn	20dn

Table-12: Most affected countries when under achievement approach in HDR 1997 (Atkinson based formulation is replaced with Chaubey's formulation)

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Georgia	23up	20up	17up	Saudi Arabia	25dn	32dn	15dn
Luxembourg	18up	13up	18up	Oman	25dn	32dn	14dn
Armenia	16up	19up	17up	Algeria	25dn	23dn	24dn
Ajjerbaijan	15up	22up	12up	Libyan Arab Jamah	23dn	30dn	15dn
Kuwait	15up	12up	17up	Brazil	21dn	30dn	13dn
Ukraine	14up	13up	9up	Iran	21dn	19dn	19dn

Table-13: Most affected countries when under achievement approach in HDR 1998 (Atkinson based formulation is replaced with Chaubey's formulation)

6. Impact of Bhatnagar's Alternative Multi-Step Utility Function Using Kakwani's Achievement Approach

Following Bhatnagar (2001-b), the adjusted values from the alternative Multi-step formulation can be obtained as follows:

$$\text{Adjusted Real GDP}_{proposed} = y, \quad 0 < y \leq y^* \qquad (10A)$$

$$\text{Adjusted Real GDP}_{proposed} = y^* + y^* \log(y/y^*), \quad y^* < y \leq 2y^* \qquad (10B)$$

$$\text{Adjusted Real GDP}_{proposed} = y^* + (1/2)y^*(\log 2) + (1/2)y^* \log(y/y^*),$$
$$\text{when } 2y^* < y \leq 4y^* \qquad (10C)$$

$$\text{Adjusted Real GDP}_{proposed} = y^* + (1/4)y^*(2\log 2 + \log 4) + (1/4)y^* \log(y/y^*),$$
$$\text{when } 4y^* < y \leq 6y^* \qquad (10D)$$

$$\text{Adjusted Real GDP}_{proposed} = y^* + (1/8)y^*(4\log 2 + 2\log 4 + \log 6)$$
$$+ (1/8)y^* \log(y/y^*), \quad \text{when } 6y^* < y \leq 8y^* \qquad (10E)$$

In general,

$$\text{Adjusted Real GDP}_{proposed} = (\sum_{m=0}^{n} (1/2^m) y^* \log(2m)) + ((1/2^n)y^* \log(y/y^*)),$$
$$\text{for } 2ny^* < y \leq 2(n+1)y^* \text{ and } n \geq 1 \qquad (10F)$$

When any real GDP per capita income level in the HDR of 1995 to 1998 is adjusted using alternative multi-step formulation, the values of 'M' and' m' corresponding to maximum and minimum goalposts are found to be:

Year	Threshold income (in PPP$)	m (in PPP$)	M (in PPP$)
1998	5990	100	8993
1997	5835	100	8769
1996	5711	100	8689
1995	5120	100	7731

Table-14: Maximum and Minimum values for income component following Bhatnagar's alternative formulation

Following Kakwani's Achievement approach, the GDP achievement indicator can be expressed as

$$I_{A(bhatnagar)} = 1 - \{(\text{Adjusted Max. GDP}_{bhatnagar} - \text{Adjusted actual GDP}_{bhatnagar}) / (\text{Adjusted Max. GDP}_{bhatnagar} - 100)\}^{(1-\epsilon)} \qquad (11)$$

Using (11), the value for achievement indicator for the income component while adopting the Multi-step alternative formulation for say HDR1997, can be computed as

$$I_{A(bhatnagar)} = 1 - \{(8769 - \text{adjusted actual GDP}_{bhatnagar}) / 8669\}^{(1-\epsilon)} \qquad (12)$$

Like before, the life expectancy at birth Achievement Indicator and Education Achievement Indicator due to Kakwani's approach can be computed by Equation (3) and (4) respectively. The value of fresh HDI under the premises of Multi-step alternative formulation using Kakwani's Achievement indicator approach would be the average of L_A, E_A and $I_{A(bhatnagar)}$ for each country.

The effect in the ranks of countries due to the Multi-step formulation proposed by Bhatnagar (2001-b) using Kakwani's Achievement Approach has been studied for HDR data from 1995 to 1998 by considering the three illustrative values of 'ϵ'. The

most affected countries due to Kakwani's Achievement Approach when alternative Multi-step formulation due to Bhatnagar (2001-b) is used instead of Atkinson based multi-step for utility function are observed to be as follows:

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Qatar	25up	25up	24up	Botswana	26dn	27dn	20dn
Georgia	24up	32up	19up	Tunisia	24dn	25dn	20dn
Armenia	21up	26up	18up	Iran	23dn	28dn	20dn
UAE	16up	17up	16up	Turkey	20dn	22dn	15dn
Brunei Darussalam	14up	14up	14up	Brazil	20dn	23dn	13dn
St. Vincent	14up	17up	11up	Costa Rica	20dn	21dn	18dn

Table-15: Most affected countries under bhatnagar's alternative utility function adopting Kakwani's approach in HDR 1995 as against Traditional UNDP multi-step formulation

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Armenia	29up	34up	20up	Algeria	30dn	31dn	26dn
Qatar	17up	19up	16up	Iran	30dn	30dn	23dn
Kuwait	17up	18up	16up	Botswana	27dn	28dn	23dn
Grenada	16up	17up	13up	Tunisia	23dn	24dn	21dn
Cuba	16up	15up	14up	Jordan	19dn	23dn	12dn
Lithuania	16up	18up	13up	Libyan Arab Jamah	18dn	22dn	10dn

Table-16: Most affected countries under bhatnagar's alternative utility function adopting Kakwani's approach in HDR 1996 as against Traditional UNDP multi-step formulation

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Armenia	21up	29up	14up	Algeria	22dn	24dn	19dn
Kuwait	20up	22up	20up	Syrian Arab Rep.	22dn	24dn	15dn
Luxembourg	19up	24up	13up	Tunisia	21dn	23dn	15dn
Latvia	17up	26up	12up	Iran	20dn	28dn	15dn
Brunei Darussalam	15up	15up	13up	Brazil	16dn	22dn	11dn
Mauritius	15up	13up	16up	Botswana	15dn	15dn	14dn

Table-17: Most affected countries under bhatnagar's alternative utility function adopting Kakwani's approach in HDR 1997 as against Traditional UNDP multi-step formulation

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Kuwait	28up	30up	28up	Botswana	13dn	13dn	20dn
Brunei Darussalam	25up	29up	25up	Spain	13dn	15dn	7dn
Luxembourg	22up	25up	22up	New Zealand	13dn	13dn	4dn
Qatar	22up	24up	22up	Fin land	13dn	15dn	8dn
Mauritius	20up	21up	20up	Costa Rica	11dn	11dn	6dn

Table-18: Most affected countries under bhatnagar's alternative utility function adopting Kakwani's approach in HDR 1998 as against Traditional UNDP multi-step formulation

When the alternative Multi-step formulation of the Utility function is used as a replacement of the Atkinson based Multi-step utility function and Kakwani's Achievement Approach is followed in both the cases, variations in the ranks of the countries have further been studied. The most affected countries whose ranks have either gone up or moved down on account of applying Bhatnagar's alternative formulation instead of Atkinson based utility function under Kakwani's Achievement Indicator approach in both the cases are as under:

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Qatar	22up	20up	21up	Tunisia	23dn	24dn	19dn
UAE	17up	17up	17up	Botswana	23dn	24dn	17dn
Georgia	15up	18up	14up	Turkey	21dn	22dn	16dn
Bahrain	14up	12up	14up	Iran	21dn	25dn	18dn
Armenia	13up	15up	13up	Brazil	20dn	23dn	13dn
Brunei Darussalam	13up	14up	13up	Syrian Arab Rep.	18dn	17dn	15dn

Table-19: Most affected countries in HDR 1995 when under achievement approach Atkinson based formulation is replaced by Bhatnagar's alternative formulation

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Qatar	15up	15up	14up	Algeria	29dn	27dn	27dn
Armenia	14up	12up	12up	Iran	24dn	21dn	22dn
Kuwait	13up	10up	12up	Libyan Arab Jamah	19dn	22dn	11dn
UAE	12up	12up	13up	Botswana	19dn	16dn	16dn
Korean DPR	11up	9up	7up	Tunisia	15dn	13dn	16dn
Samoa (western)	11up	7up	7up	Oman	14dn	20dn	5dn

Table-20: Most affected countries in HDR 1996 when under achievement approach Atkinson based formulation is replaced by Bhatnagar's alternative formulation

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Kuwait	14up	15up	16up	Iran	20dn	29dn	15dn
Luxembourg	13up	13up	8up	Syrian Arab Rep.	18dn	17dn	14dn
Armenia	12up	14up	8up	Algeria	18dn	17dn	16dn
Cuba	12up	12up	10up	Libyan Arab Rep.	17dn	21dn	10dn
Georgia	10up	15up	4up	Tunisia	17dn	17dn	14dn

Table-21: Most affected countries in HDR 1997 when under achievement approach Atkinson based formulation is replaced by Bhatnagar's alternative formulation

Countries	Effect on Ranks when			Countries	Effect on Ranks when		
	$\in$ =1/2	$\in$ =2/3	$\in$ =1/3		$\in$ =1/2	$\in$ =2/3	$\in$ =1/3
Luxembourg	15up	7up	15up	Botswana	20dn	24dn	20dn
Brunei Darussalam	15up	10up	15up	Algeria	15dn	18dn	15dn
Cuba	13up	14up	13up	Iran	12dn	12dn	12dn
Georgia	10up	10up	10up	Syrian Arab Rep.	11dn	12dn	11dn
Qatar	10up	6up	10up	Tunisia	10dn	11dn	10dn
Mauritius	9up	6up	9up	Indonesia	9dn	9dn	9dn

Table-22: Most affected countries in HDR 1998 when under achievement approach Atkinson based formulation is replaced by Bhatnagar's alternative formulation

7. Conclusion

Corresponding to the three methods of 'discounting' the income component of Human Development Index available to us under Multi-step formulation viz., Atkinson-based multi-step utility function, Chaubey(1998)'s Two-step utility function and Bhatnagar(2001-b)'s alternative Multi-step formulation, one can generate six independent sets of ranking depending whether Kakwani's achievement approach is considered or not. Due to prime considerations of relative importance and avoiding any redundancy, we have restricted ourselves in Sections from 3 to 6 of the paper to study of impacts of sensitization in ranks of the countries under specific scenario only. However, as a mater of academic interest, one can further study variations in ranks of the countries under following situations:

(a)   when under Kakwani's achievement approach, Chaubey(1998)'s Two-step utility function is replaced by the alternative Multi-step utility function proposed by Bhatnagar (2001-b);

(b)   achievement approach is replaced by Chaubey's Two-step utility function under Kakwani's Achievement approach

(c)   when Chaubey(1998)'s Two-step utility function without Kakwani's Achievement approach is replaced by Bhatnagar(2001-b)'s alternative Multi-step formulation under Kakwani's Achievement approach

(d)   when Bhatnagar(2001-b)'s proposed Multi-step alternative formulation without Kakwani's Achievement approach is replaced by the same utility function under Kakwani's Achievement approach.

For different values of the parameter '$\in$', we obtain identical conclusions regarding the direction (upward/ downward) in which the rank of any country would be affected by use of Kakwani's Achievement Indicator approach while formulating the Indices of Life Expectancy at birth, Educational attainments and real GDP per capita. For indicative purposes, one may therefore, for the sake of simplicity adopt '$\in$' as ½ which is the middle value of the entire parametric range.

The Tables-1 to 22 summarize, the impacts Kakwani's Achievement Indicator approach on ranks of the countries which have been most affected amongst those covered in the UNDP's HDRs for the period 1995 to 1998, under varying formulations of Income-transformation function.

Acknowledgement

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