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# ANALYSIS OF RELIABILITY CHARACTERISTICS OF A COMPLEX ENGINEERING SYSTEM UNDER COPULA

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## Abstract

This paper models a complex system to analyse its reliability characteristics. The system has two possible modes-normal and failed with two types of repair facilities-major and minor. The system can fail due to failure of any units which can fail in n-mutually exclusive ways or common cause failure. Each component of the system has a constant failure rate. The system can be repaired with two different distributions viz. exponential and arbitrary. By employing supplementary variable technique and Gumbel- Hougaard family copula, Laplace transformation of various transition state probabilities, availability and cost analysis (expected profit) along with steady-state behaviour of the system and their plots have been obtained. Numerical examples with a way to highlight the important results have been appended at last.

**Key Words:** Common Cause Failure, Complex System, Reliability, Availability, Cost Analysis, Supplementary Variable, Gumbel-Hougaard Family Copula.

## 1. Introduction

The model similar to subject under study can be found in references [1, 3]. These studies however do not incorporate the concept of copula applicable for a joint distribution when two different type of repair possible between adjacent states which is a possibility in physical systems. In references [2, 4, 5] copulas have been employed in modelling and discussed variety of its applications and their choice in multivariate environmental data. [7] applied the Gumbel-Hougaard family copula in a parallel redundant complex system with two types of failure under preemptive-resume repair discipline and found the improvement in results of reliability measures.

The present paper applied the features of Gumbel-Hougaard family of copula to develop a mathematical model when two different distributions are possible in repair between two adjacent states which was not considered in [1, 3]. The model consists of a multi-component automatic system which can fail due to common cause (i.e. all the components fail simultaneously) or due to failure of any one of the n-components in n-mutually exclusive ways. If the system is in any failure modes it may require major or minor repair depending upon type of failures. When the system fails completely due to common cause, it is repaired with two ways namely exponential and arbitrary to reach its normal state directly. So, in this model authors tried to address the problem where two different repair facilities namely exponential and arbitrary are available between adjacent states $S_{cc}$ and $S_0$ (where $S_0$ is the normal state and $S_{cc}$ is the completely failed state due to common cause failure). Each component of the system has a constant failure rate. These rates vary from component to component as all the n-components of the unit are of different types. The system is studied by using the supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula to obtain various reliability measures such as transition state probabilities, steady state probability, availability and cost analysis. At last some particular cases of the system are taken to highlight the different possibilities. Transition diagram for this model is shown in Figure 0.

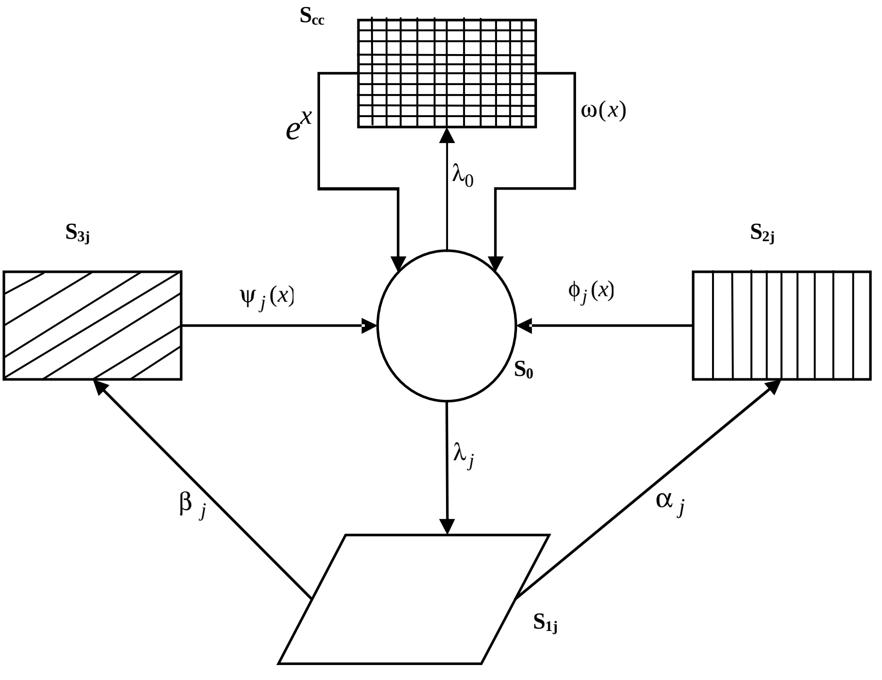


**Figure 0:** State Transition Diagram

## 2. Brief Introduction of Copulas

Nelsen, R. B. [6] discussed the theory copulas. The joint distribution function implicitly contains a description of the dependence structure of a random vector and, from one point of view, copulas are functions that join or "couple" multivariate distribution functions to their fixed one-dimensional distribution functions. In other words, copulas are multivariate distribution functions whose one-dimensional margins are uniform on the interval [0, 1]. The copula approach is very natural when a complex system repaired by couple of ways.

**Definition (i) (Copula)** A d-dimensional copula is a distribution function on $[0, 1]^d$ with standard uniform marginal distributions. Let C (u) = C($u_1$, ..., $u_d$) be the distribution functions which are copulas. Hence C is a mapping of the form C: $[0, 1]^d \rightarrow [0, 1]$, i.e. a mapping of the unit hypercube into the unit interval. The following three properties must hold:

1. C($u_1$, ..., $u_d$) is increasing in each component $u_i$.
2. C(1, ...1, $u_i$, 1, ..., 1) = $u_i$ for all $i \in \{1, ...d\}$, $u_i \in [0, 1]$.
3. For all ($a_1$,..., $a_d$), ($b_1$,..., $b_d$)$\in$ [0, 1] with $a_i \leq b_i$ we have:

$$\sum_{i_1=1}^{2} \cdots\cdots \sum_{i_1=1}^{2} (-1)^{i_1 + \cdots\cdots + i_d} C(u_{i_1,\ldots\ldots,} \; u_{i_d}) \geq 0$$

where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in \{1, ..., d\}$.

**Theorem (Sklar)** Let F be a joint distribution function with margins $F_1,\ldots\ldots,F_d$ (not necessarily continuous). Then there exists a copula C: $[0, 1]^d \rightarrow [0, 1]$, such that for all $x_1,\ldots\ldots,x_d$ in $\overline{\Re} = [-\infty, \infty]$

$$F(x_1, \ldots\ldots, x_d) = C(F_1(x_1),\ldots\ldots,F_d(x_d)) \qquad \text{(i)}$$

If the margins are continuous then C is unique; otherwise C is uniquely determined on Ran $F_1 \times \ldots \times$ Ran $F_d$, where Ran $F_i$ denotes the range of $F_i$ : Ran $F_i = F_i(\overline{\Re})$. Conversely, if C is a copula and $F_i,\ldots, F_d$ are distribution functions, then the function F defined in (i) is a joint distribution function with margins $F_i,\ldots, F_d$.

**Definition (ii)** If F is a joint distribution function with marginals $F_1,\ldots, F_d$ and theorem (Sklar) holds, we say that C is a copula of F (or a random vector X ~ F ). If the marginals are continuous then C is the unique copula of F (or X).

The copula is the distribution function of the componentwise probability transformed random vector. Alternatively, we can evaluate (i) at the arguments $x_i = F_i(u_i)$, $0 \le u_i \le 1$, i=1,...,d, and use the property of the generalised inverse to obtain

$$C(u_1\ldots\ldots u_d) = F(F_1^{\leftarrow}(u_1),\ldots\ldots,F_d^{\leftarrow}(u_d)), \qquad \text{(ii)}$$

where $F^{\leftarrow}$ is the generalised inverse of F .

## Copula Families

There are some main family of copulas.

● **Archimedian copulas**

The family of Archimedean copulas has been studied extensively by a number of authors including [3] and [6]. Well known representatives of the Archimedean family are the Gumbel-Hougaard, Frank and Clayton copulas.

***Bivariate Clayton copula***

$$C_\theta(u_1, u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta} \qquad \text{(iii)}$$

The Clayton copula is well defined for $0 < \theta < \infty$ and for $\theta \rightarrow 0$ and $\theta \rightarrow \infty$ it converges to the product copula and comonotonicity respectively. Sometimes it is referred to as the Cook-Johnson copula or the Pareto family of copulas. Due to its property of lower tail dependence, the Clayton copula is a possible candidate for model building in the financial context.

***Bivariate Gumbel-Hougaard family copula***

$$C_\theta(u_1, u_2) = \exp(-((-\log u_1)^\theta + (-\log u_2)^\theta)^{1/\theta}), \quad 1 \le \theta \le \infty \qquad \text{(iv)}$$

For $\theta = 1$ the Gumbel- Hougaard copula models independence, for $\theta \rightarrow \infty$ it converges to comonotonicity.

***Bivariate Frank copula***

$$C_\theta(u_1, u_2) = -\frac{1}{\theta}\log\left[1 + \frac{(\exp(-\theta u_1) - 1) - (\exp(-\theta u_2) - 1)}{\exp(-\theta) - 1}\right], \quad \theta \in \Re \qquad \text{(v)}$$

● **Marshall-Olkin copula**

The Marshall-Olkin copula family has the attractive feature that it may be derived from a simple stochastic process model called a common Poisson shock model. For more detail one can study of [6].

● **Elliptical Copulas**

A unique copula is implicit in every multivariate distribution with continuous marginals, and useful classes of parametric copulas are those implicit in elliptical distributions. These copulas have the virtue that they extend to arbitrary dimensions and are rich in parameters, which facilitates their fitting to data. The Gaussian and t-copulas are defined as

$$C_{\Sigma}^{Ga}(u) = \Phi_{\Sigma}\left((\Phi^{-1}(u_1),......, \Phi^{-1}(u_d)\right) \quad \text{(vi)}$$

$$C_{\nu,\Sigma}^{t}(u) = t_{\nu,\Sigma}\left(t_{\nu}^{-1}(u_1),......, t_{\nu}^{-1}(u_d)\right) \quad \text{(vii)}$$

where $\Phi_{\Sigma}$ and $t_{\nu,\Sigma}$ denote the joint distribution functions of a standard d-dimensional normal random vector with covariance matrix $\Sigma$ and a d-dimensional multivariate t-distribution with $\nu$ degrees of freedom and correlation matrix R respectively. The t-copula is especially appealing, because at the cost of only one extra parameter $\nu$ we get a flexible family of copulas suitable for high-dimensional modeling that includes the Gaussian copula as a special limiting case.

## Application of Copulas in this Study

In this paper authors applied a useful application of Gumbel- Hougaard family copula in the repair of a failed system. We focus this area for the reason: the system can be repaired by two different ways. Authors believe that the interactive study of this should be enhancing the availability of the system.

## Assumptions

1) Initially the system is in normal state.
2) The system has two states: normal and failed.
3) The each component of the system has a constant failure rate and an arbitrary repair rate.
4) These rates vary from component to component as all the n-components of the unit are of different types.
5) The system is repaired from failed state after detecting the type of repair viz., major or minor.
6) Transition from state $S_0$ to state $S_{1j}$ follows two different distributions.
7) After repairing system is as good as new. Repair never damages anything.
8) System states are: normal ($S_0$), failed ($S_{1j}$), major repair ($S_{2j}$), minor repair ($S_{3j}$), j=1, 2 …n and common cause ($S_{cc}$).
9) Joint probability distribution of failure rate from state $S_0$ to the state $S_{1j}$ computed by Gumbel- Hougaard family of Copula.

## Notations

The following notations are associated with this model:

| | |
|---|---|
| $\phi_j(x), \zeta_j(x)$ | Rates of major repair and corresponding pdf of repair times respectively. |
| $\psi_j(x), \xi_j(x)$ | Rates of minor repair and corresponding pdf of repair times respectively. |
| $\omega(x), \chi(x)$ | Repair rate for common cause failure and corresponding pdf of repair times respectively. |
| $\lambda_0, \lambda_j$ | Constant rate of transition from state $S_0$ to $S_{cc}$ or $S_{1j}$. |
| $\alpha_j, \beta_j$ | Constant detection rate of system in $S_{1j}$ being assigned to state $S_{2j}$, $S_{3j}$. |
| $P_k(t)$ | P[at epoch t the system is in state $S_k$ ]; k=0, cc, ij; i=1,2,3, j=1,2,…..n. |

| | |
|---|---|
| $P_h(x, t)dx$ | P [system is in state $S_h$ at epoch t and has sojourned in this state for duration between x and x+dx]: h=2j, 3j, cc; j=1,2,…..n. |
| $u_1, u_2$ | Marginal distribution of random variables, where $u_1 = \log(x)$ and $u_2 = \lambda_j$ . |
| $m_j$, $M_j$ | Expected duration of minor/major repair of the total failed unit. |
| Σ | Sum over from 1 to n unless otherwise mentioned. |
| $E_p(t)$ | Expected profit during the interval (0, t]. |
| $K_1$, $K_2$ | Revenue per unit time and service cost per unit time respectively. |
| $u_1, u_2$ | Marginal distribution of random variables, where $u_1 = e^x$ and $u_2 = \omega(x)$ . |

Letting $u_1 = e^x$ and $u_2 = \omega(x)$, the expression for joint probability (failed state $S_{cc}$ to normal $S_0$) according to Gumbel-Hougaard family is given as:

$$\exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}$$

## Formulation of Mathematical Model

By elementary probability and continuity arguments, we can obtain the following set of difference-differential equations which is continuous in time and discrete in space for the present mathematical model

$$\left[\frac{\partial}{\partial t} + \lambda_0 + \Sigma\lambda_j\right]P_0(t) = \int_0^\infty \left(\Sigma P_{2j}(x,t)\phi_j(x)\right)dx + \int_0^\infty \left(\Sigma P_{3j}(x,t)\psi_j(x)\right)dx$$
$$+ \int_0^\infty P_{cc}(x,t)\exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}\, dx \qquad (1)$$

$$\left[\frac{\partial}{\partial t} + \alpha_j + \beta_j\right]P_{1j}(t) = \lambda_j P_0(t) \qquad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi_j(x)\right]P_{2j}(x,t) = 0 \qquad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \psi_j(x)\right]P_{3j}(x,t) = 0 \qquad (4)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}\right]P_{cc}(t) = 0 \qquad (5)$$

### Boundary Conditions

$$P_{2j}(0,t) = \alpha_j P_{1j}(t) \qquad (6)$$

$$P_{3j}(0,t) = \beta_j P_{1j}(t) \qquad (7)$$

$$P_{cc}(0,t) = \lambda_0 P_0(t) \qquad (8)$$

## Initial Conditions

$P_0(0) = 1$ and other state probabilities are zero at t = 0 (9)

## Solution of the Model

Taking Laplace transformation of (1 - 8) and using (9), we obtain

$$\left[s+\lambda_0+\Sigma\lambda_j\right]\overline{P}_0(s) = 1+\int_0^\infty\left(\Sigma\,\overline{P}_{2j}(x,s)\phi_j(x)\right)dx+\int_0^\infty\left(\Sigma\,\overline{P}_{3j}(x,s)\psi_j(x)\right)dx$$

$$+\int_0^\infty \overline{P}_{cc}(x,s)\exp[x^\theta+\{\log\omega(x)\}^\theta]^{1/\theta}\,dx \quad (10)$$

$$\left[s+\alpha_j+\beta_j\right]\overline{P}_{1j}(s) = \lambda_j\overline{P}_0(s) \quad (11)$$

$$\left[s+\frac{\partial}{\partial x}+\phi_j(x)\right]\overline{P}_{2j}(x,s) = 0 \quad (12)$$

$$\left[s+\frac{\partial}{\partial x}+\psi_j(x)\right]\overline{P}_{3j}(x,s) = 0 \quad (13)$$

$$\left[s+\frac{\partial}{\partial x}+\exp[x^\theta+\{\log\omega(x)\}^\theta]^{1/\theta}\right]\overline{P}_{cc}(s) = 0 \quad (14)$$

$$\overline{P}_{2j}(0,s) = \alpha_j\overline{P}_{1j}(s) \quad (15)$$

$$\overline{P}_{3j}(0,s) = \beta_j\overline{P}_{1j}(s) \quad (16)$$

$$\overline{P}_{cc}(0,s) = \lambda_0\overline{P}_0(s) \quad (17)$$

Solving (10 - 14) with the help of (15 - 17), we get

$$\overline{P}_0(s) = \left[s+\sum\lambda_j\left(1-\frac{\alpha_j\overline{\xi}_j(s)+\beta_j\overline{\zeta}_j(s)}{s+\alpha_j+\beta_j}\right)+\lambda_0(1-\overline{\chi}(x)\right]^{-1} \quad (18)$$

$$\overline{P}_{1j}(s) = \frac{\lambda_j}{s+\alpha_j+\beta_j}\overline{P}_0(s) \quad (19)$$

$$\overline{P}_{2j}(s) = \left[\frac{\alpha_j\lambda_j}{s(s+\alpha_j+\beta_j)}\right][1-\overline{\xi}_j(s)]\overline{P}_0(s) \quad (20)$$

$$\overline{P}_{3j}(s) = \left[\frac{\beta_j\lambda_j}{s(s+\alpha_j+\beta_j)}\right][1-\overline{\zeta}_j(s)]\overline{P}_0(s) \quad (21)$$

$$\overline{P}_{cc}(s) = \lambda_0\left[\frac{1-\overline{\chi}(s)}{s}\right]\overline{P}_0(s) \quad (22)$$

## Evaluation of Laplace Transformation of Up and Down State Probabilities

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s) = \overline{P}_0(s) \tag{23}$$

$$\overline{P}_{failed}(s) = \overline{P}_{1j}(s) + \overline{P}_{2j}(s) + \overline{P}_{3j}(s) + \overline{P}_{cc}(s) \tag{24}$$

Also, it is worth noticing that

$$\overline{P}_{up}(s) + \overline{P}_{failed}(s) = \frac{1}{s} \tag{25}$$

## Asymptotic Behaviour of the System

Using Abel's lemma in Laplace transformation, viz.

$\lim_{s\to 0}\{s\overline{F}(s)\} = \lim_{t\to\infty} F(t) = F(say)$,

provided the limit on right hand exist in (18) through (21) the following time independent probabilities are obtained

$$P_0 = \left[1 + \lambda_0 z + \sum\left(1 + \frac{\alpha_j m_j + \beta_j M_j}{\alpha_j + \beta_j}\right)\lambda_j\right]^{-1} \tag{26}$$

$$P_{1j} = \frac{\lambda_j}{\alpha_j + \beta_j} P_0 \tag{27}$$

$$P_{2j} = \frac{\alpha_j \lambda_j m_j}{\alpha_j + \beta_j} P_0 \tag{28}$$

$$P_{3j} = \frac{\beta_j \lambda_j M_j}{\alpha_j + \beta_j} P_0 \tag{29}$$

Where $m_j$ and $M_j$ are expected duration of minor/major repair of the total failed unit i.e.

$$m_j = \int_0^\infty x\xi_j(x)dx,\ M_j = \int_0^\infty x\zeta_j(x)dx \quad and \quad z = \int_0^\infty x\chi(x)dx$$

The asymptotic state availability of the system is

$$P_{up} = P_0 \tag{30}$$

## Particular Cases

Assuming that repair follows exponential distribution, setting

$$\overline{\xi}_j(s) = \frac{\phi_j}{s+\phi_j}, \overline{\zeta}_j(s) = \frac{\psi_j}{s+\psi_j} \text{ and } \overline{\chi}_j(s) = \frac{\exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}}$$

in (18) through (22), the Laplace transformations of various state probabilities are as follows:

$$\overline{P}_0(s) = \overline{P}_{up} = \left[s\left(1 + \frac{\lambda_0}{s + \exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}}\right) + \sum \frac{\lambda_j\{(s+\alpha_j+\phi_j)(s+\psi_j) + \beta_j(s+\phi_j)\}s}{(s+\phi_j)(s+\psi_j)(s+\alpha_j+\beta_j)}\right]^{-1} \tag{31}$$

$$\overline{P}_{1j}(s) = \frac{\lambda_j}{s+\alpha_j+\beta_j}\overline{P}_0(s) \tag{32}$$

$$\overline{P}_{2j}(s) = \frac{\alpha_j\lambda_j}{(s+\phi_j)(s+\alpha_j+\beta_j)}\overline{P}_0(s) \tag{33}$$

$$\overline{P}_{3j}(s) = \frac{\beta_j\lambda_j}{(s+\psi_j)(s+\alpha_j+\beta_j)}\overline{P}_0(s) \tag{34}$$

$$\overline{P}_{cc}(s) = \frac{\lambda_0}{s+\exp[x^\theta+\{\log\omega(x)\}^\theta]^{1/\theta}}\overline{P}_0(s) \tag{35}$$

## Numerical Computations

### (i) Availability Analysis:

(a) Taking $\lambda_0 = \lambda_j = \alpha_j = \beta_j = 0.05, \phi_j = \psi_j = \omega = 1, x = 1, \theta = 1$ and n=10. Setting

$\overline{\xi}_j(s) = \dfrac{\phi_j}{s+\phi_j}$, $\overline{\zeta}_j(s) = \dfrac{\psi_j}{s+\psi_j}$ and $\overline{\chi}_j(s) = \dfrac{\exp[x^\theta+\{\log\omega(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\omega(x)\}^\theta]^{1/\theta}}$ in (31) and taking inverse Laplace transform, one may have

$$P_{up}(t) = 0.026393\, e^{(-2.7789t)} + 0.82019\, e^{(-0.79465t)} \cos(0.078278t) + 2.2371\, e^{(-0.79465t)} \sin(0.078278t) + 0.15341 \tag{36}$$

(b)Setting $\lambda_0 = \lambda_j = \alpha_j = \beta_j = 0.25, \phi_j = \psi_j = \omega = 1, x = 1, \theta = 1, n = 10$ and considering that system follow exponential distribution then from (31) and taking inverse Laplace transform, one can get (37)

$$P_{up}(t) = 0.48633\, e^{(-3.4765t)} + 0.39729\, e^{(-1.7458t)} \cos(0.55765t) - 0.65574\, e^{(-1.7458t)} \sin(0.55765t) + 0.11629 \tag{37}$$

(c) When repair follow exponential distribution and various parameters are fixed as x =1, θ=1, $\alpha_j = 0.10, \beta_j = 0.05, \phi_j = \psi_j = \omega = 1, \lambda_0 = 0.15, \lambda_j = 0.20$ and n=10. Substituting these values of parameters in (31) and taking inverse Laplace transform, we have

$$P_{up}(t) = 0.28593\, e^{(-3.1192t)} + 0.65306\, e^{(-1.4495t)} \cos(0.20291t) - 1.5353 e^{(1.4495t)} \sin(0.20291t) + 0.061018 \tag{38}$$

In (36), (37) and (38), setting t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 one may obtain Tables 1, 2 and 3 respectively. These tables demonstrate how availability of the system changes with respect to the passage of time.

| Time | $P_{up}$ |
|---|---|
| 0 | 1.00000 |
| 1 | 0.60345 |
| 2 | 0.39001 |
| 3 | 0.27493 |
| 4 | 0.21460 |
| 5 | 0.18372 |
| 6 | 0.16822 |
| 7 | 0.16057 |
| 8 | 0.16057 |
| 9 | 0.15503 |
| 10 | 0.15417 |

Table 1: Time vs. Availability

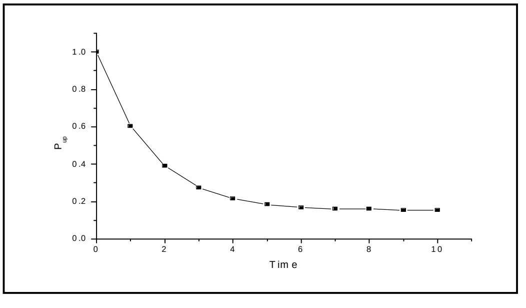


Figure 1. Time vs. Availability

| Time | $P_{up}$ |
|---|---|
| 0 | 1.0000000 |
| 1 | 0. 1296953 |
| 2 | 0. 1042442 |
| 3 | 0. 1127226 |
| 4 | 0. 1156841 |
| 5 | 0. 1162929 |
| 6 | 0. 1163827 |
| 7 | 0. 1163908 |
| 8 | 0. 1163904 |
| 9 | 0. 1163901 |
| 10 | 0. 1163900 |

Table 2: Time vs. Availability

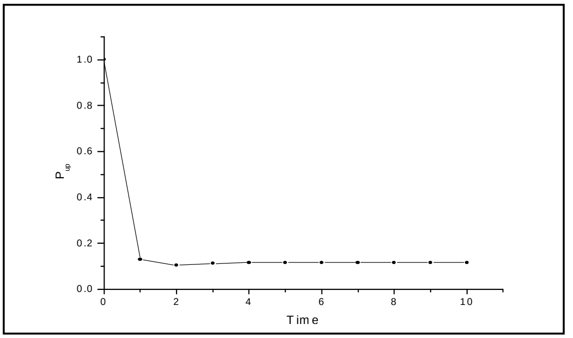


Figure 2. Time vs. Availability

| Time | $P_{up}$ |
|---|---|
| 0 | 1.00000 |
| 1 | 0.15116 |
| 2 | 0.06124 |
| 3 | 0.05661 |
| 4 | 0.05900 |
| 5 | 0.06033 |
| 6 | 0.06081 |
| 7 | 0.06096 |
| 8 | 0.06100 |
| 9 | 0.06101 |
| 10 | 0.06101 |

Table 3: Time vs. Availability

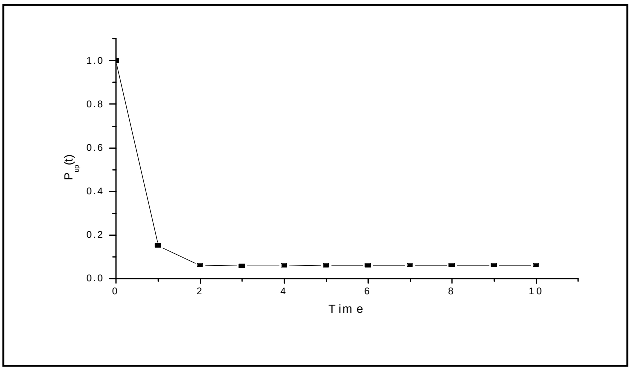


Figure 3. Time vs. Availability

**(iii) Mean-Time-to-Failure (M.T.T.F.):**

Setting $\alpha_j = \beta_j = 0$ in (18) and taking limit as s tends to zero, the MTTF can be obtained as

$$M.T.T.F. = \lim_{s \to 0} \overline{P}_{up}(s) = \frac{1}{\lambda_0 + \sum \lambda_j} \tag{39}$$

Setting $\lambda_0 = 0.25$, n=10 and varying $\lambda_j$ as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 in (39) one may obtain Table 4 which demonstrates variation of MTTF with respect to $\lambda_j$.

Further setting $\lambda_j = 0.25$, n=10 and varying $\lambda_0$ as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 in (39) we may get Table 5 which gives value of MTTF with respect to the change in $\lambda_0$.

| $\lambda_j$ | MTTF |
|---|---|
| 0.10 | 0.80000 |
| 0.20 | 0.44444 |
| 0.30 | 0.30769 |
| 0.40 | 0.23529 |
| 0.50 | 0.19047 |
| 0.60 | 0.16000 |
| 0.70 | 0.13793 |
| 0.80 | 0.12121 |
| 0.90 | 0.10810 |

Table 4: $\lambda_j$ vs. MTTF

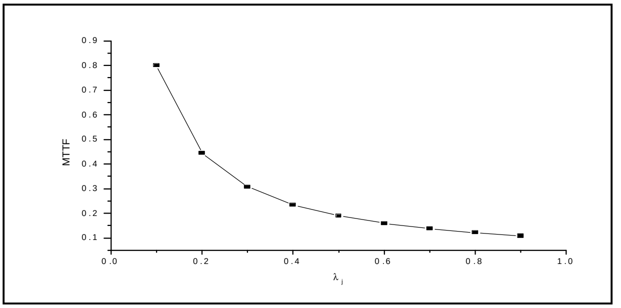


Figure 4: $\lambda_j$ vs. MTTF

| $\lambda_0$ | MTTF |
|---|---|
| 0.10 | 0.38461 |
| 0.20 | 0.37037 |
| 0.30 | 0.35714 |
| 0.40 | 0.34482 |
| 0.50 | 0.33333 |
| 0.60 | 0.32258 |
| 0.70 | 0.31250 |
| 0.80 | 0.30303 |
| 0.90 | 0.29411 |

Table5: $\lambda_0$ vs. MTTF

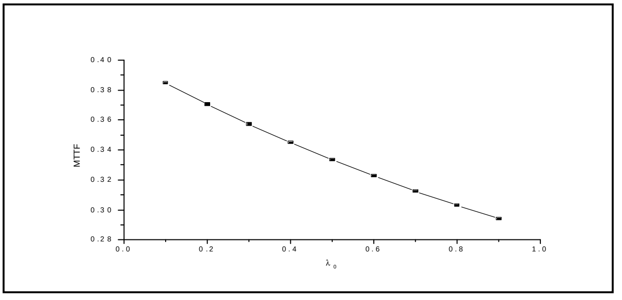


Figure 5: $\lambda_0$ vs. MTTF

**(ii) Cost Analysis:**

(a) Taking $\lambda_0 = \lambda_j = \alpha_j = \beta_j = 0.05, \phi_j = \psi_j = \omega = 1, x = 1, \theta = 1, n = 10$ and assuming that repair follow exponential distribution then setting

$\bar{\xi}_j(s) = \dfrac{\phi_j}{s+\phi_j}$, $\bar{\zeta}_j(s) = \dfrac{\psi_j}{s+\psi_j}$ and $\bar{\chi}_j(s) = \dfrac{\exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta + \{\log\omega(x)\}^\theta]^{1/\theta}}$ in (31) and taking inverse Laplace transform, we have (36).

Let the service facility be always available, then expected profit during the interval (0, t] is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{40}$$

Where $K_1$ and $K_2$ are the revenue per unit time and service cost per unit time respectively.

Using (36) in (40) for the same set of parameters, one can obtain (41).

$$E_p(t) = K_1 [-0.009497642952\, e^{(-2.7789t)} - 1.29687076\, e^{(-0.79465t)} \cos(0.078278\, t) - 2.687451772\, e^{(-0.79465t)} \sin(0.078278\, t) + 0.15341\, t + 1.306368403] - K_2 t \tag{41}$$

(b) When repair follow exponential distribution and various parameters at somewhat higher value i.e. $\lambda_0 = \lambda_j = \alpha_j = \beta_j = 0.25, \quad \phi_j = \psi_j = \omega = 1, \quad x = 1, \theta = 1$ and n=10. Putting these values in (31) and taking inverse Laplace transform, one can obtain (37).

Using (37) in (40) same set of parameters, we get

$$E_p(t) = K_1 [-0.13989\, e^{(-3.4765t)} - 0.97629\, e^{(-1.7458t)} \cos(0.55765t) + 0.40680\, e^{(-1.7458t)} \sin(0.55765t) + 0.11629t + 0.237519] - K_2 t \quad (42)$$

Taking $K_1 = 1$; $K_2 = 0.05, 0.10, 0.15$ and using (41) and (42), the computed values of $E_p$ (t) are given in Tables 6 and 7.

| Time | $E_p(t)$ | | |
|---|---|---|---|
| | $K_2 = 0.05$ | $K_2 = 0.10$ | $K_2 = 0.15$ |
| 0 | 0 | 0 | 0 |
| 1 | 0.73020 | 0.68020 | 0.63020 |
| 2 | 1.16622 | 1.06622 | 0.96622 |
| 3 | 1.44267 | 1.29267 | 1.14267 |
| 4 | 1.63415 | 1.43415 | 1.23415 |
| 5 | 1.78158 | 1.53158 | 1.28158 |
| 6 | 1.90666 | 1.60666 | 1.30666 |
| 7 | 2.02061 | 1.67061 | 1.32061 |

Table 6: Time vs. Expected Profit

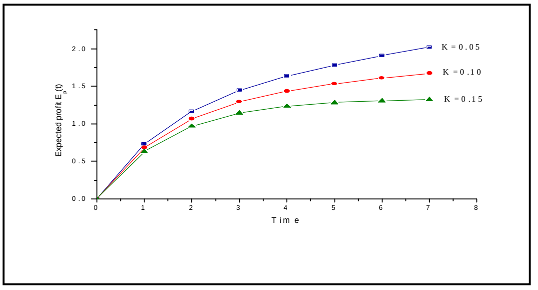


Figure 6: Time vs. Expected Profit

| Time | $E_p(t)$ | | |
|---|---|---|---|
| | $K_2 = 0.05$ | $K_2 = 0.10$ | $K_2 = 0.15$ |
| 0 | 0 | 0 | 0 |
| 1 | 0.32269 | 0.27269 | 0.22269 |
| 2 | 0.37998 | 0.27998 | 0.17998 |
| 3 | 0.43888 | 0.28888 | 0.13888 |
| 4 | 0.50543 | 0.30343 | 0.10343 |
| 5 | 0.56950 | 0.31950 | 0.06950 |
| 6 | 0.63585 | 0.33585 | 0.03585 |
| 7 | 0.70224 | 0.35224 | 0.00224 |

Table 7: Time vs. Expected Profit

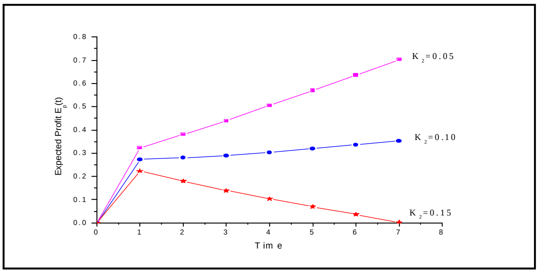


Figure 7: Time vs. Expected Profit

## Interpretation of the Result and Conclusion

From Tables 1 and 2 one can observe the variation in availability of the complex repairable system with respect to time when failure and detection rates are fixed at different values. When failure and detection rates are fixed at lower values like that $\lambda_0 = \lambda_j = \alpha_j = \beta_j = 0.05$ the availability of the system decreases with respect to time but stabilize at value 0.154 in the long run. When failure rates are fixed at 0.25, the availability of the system decreases sharply during initial stage but later on stabilizes at 0.116 in the long run. These Tables 1 and 2 and corresponding Figures 1 and 2 reveal that when the failure rate increases availability of the system decreases. Table 3 gives the availability of the system when failure and detection rates are fixed at different values. One can observe Figure 3 that availability of the system decreases sharply and attains very low value with respect to other cases but stabilizes at value 0.061in the long run.

Tables 4 and 5 yield the mean-time-to-failure (MTTF) of the system with respect to variation in $\lambda_j$ and $\lambda_0$ respectively when other parameters have been kept constant. A critical examination of the Figures 4 and 5 reveal that MTTF decreases with respect to decrement in $\lambda_j$ and $\lambda_0$ uniformly and sharply respectively but it is higher in former than the later.

When revenue cost per unit time $K_1$ fixed at 1, service cost $K_2$ varied and failure rates are kept at lower and somewhat higher values one can obtain Tables 6 and 7 for repairable system which are depicted by Figures 6 and 7 respectively. One can conclude by observing these graphs that as service cost increases, expected profit decreases. A critical examination of the graphs reveal that expected profit increases with respect to time but for the case when failure rates are kept at higher values, revenue cost per unit time fixed at one and service cost fixed at 0.15, the expected profit initially increases but later on decreases continuously. In general for lower failure rates expected profit is higher in comparison to higher failure rates.

On overall basis it is found that incorporation of copula improved the reliability of the system.

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