Journal of Reliability and Statistical Studies (ISSN: 0974-8024)
Vol. 1, Issue 1(2008) 18-24

PRELIMINARY TEST ESTIMATORS FOR THE SCALE PARAMETER OF POWER FUNCTION DISTRIBUTION

Sanjeev Kumar Sinha*, Prabhakar Singh and D. C. Singh**
Department of Statistics, Harish Chandra P.G. College,
Varanasi. India.
E-Mail: *sinha.sk70@gmail.com,**gprabhakarji@yahoo.com
Rajesh Singh***
Department of Statistics, Amravati University, Amravati, Maharastra, India.
***rsinghamt@hotmail.com

Abstract

In this paper some preliminary test estimators have been considered for estimating the scale parameter of Power function distribution when a point guess value about scale parameter is available. The shape parameter is assumed to be known. Comparisons with usual estimators in terms of relative efficiency have been made.

Key words: Power Function Distribution, Preliminary Test Estimators, Mean Square Error, Relative Efficiency.

1. Introduction:

Let X_1, $X_2, \ldots\ldots, X_n$ be a random sample from a Power function distribution having p.d.f.

$$f(x,\theta,a) = a\theta^a x^{a-1}; \quad 0 \le x \le \theta^{-1} \qquad (1.1)$$

where $\theta > 0$ and $a > 0$, are the scale and the shape parameter respectively. The maximum likelihood estimator of θ is

$\hat{\theta} = X_{(n)}^{-1} = \{\max(X_1, X_2, \ldots.. X_n)\}^{-1}$, with mean square error

$$\text{MSE}(\hat{\theta}) = 2\theta^2\{(na-1)(na-2)\}^{-1}; \; na > 2$$

where $X_{(n)}$ follows Power function distribution and $X_{(n)}^{-1}$ follows Pareto distribution with parameters θ and na. The p.d.fs of $X_{(n)}$ and $X_{(n)}^{-1}$ are respectively given as

$$f_{X_{(n)}}(x) = na\theta^{na}x^{na-1} \; ; 0 < x < \theta^{-1} \qquad \text{and} \qquad g_{\hat{\theta}}(x) = na\theta^{na}x^{-(na-1)} \; ; 0 < x < \infty$$

Some times we may have a point guess value of the parameter to be estimated. If this value is in the vicinity of the true value, the shrinkage technique is useful to get an improved estimator. Thompson (1968 a), Mehta and Srinivasan (1971), Pandey (1979a), Singh etal. (1996) and others suggested shrunken estimators for the parameters of the different distributions when a point guess value is available. They showed that these estimators perform better in terms of mean square error when the guess value is close to the true value. To resolve the uncertainty that a point guess value is approximately the true value or not, a preliminary test of significance may be employed. Using this concept, Preliminary test shrunken estimators have been considered in different contexts by Davis, R. L. and Arnold(1970), Pandey et al. (1988), Singh and Shukla (2000), Saleh (2006) Prakash, Singh and Singh (2006), Singh, Prakash and Singh (2007) and others.

2. The Preliminary Test Estimators and their Properties

From empirical studies it has been established that the shrunken estimators performs better than usual estimator when our guess value be very close to the true value of the parameter. Therefore to make sure whether θ is close to θ_0 or not, we may test H_0: $\theta = \theta_0$ against $H_1: \theta \neq \theta_0$ .The critical region for the above test is

$\omega = \left\{ X : X_{(n)} \leq \theta_0^{-1} \alpha^{\frac{1}{na}} or\ X_{(n)} \geq \theta_0^{-1} \right\}$, where α is some pre-assigned level of significance.

The shrunken estimator for parameter **θ** when some prior point guess value θ_0 for θ is available is proposed as

$$Y = \left(\hat{\theta}\right)^{k} \left(\theta_0\right)^{1-k} \tag{2.1}$$

where $0 \leq k \leq 1$ is a shrunken factor specified by the experimenter according to his belief in the guess value. The value of k near to zero indicates a strong faith in guess value $\boldsymbol{\theta_0}$ and near to one shows a strong belief in sample value. The expressions for mean square errors of the estimator defined in equation (2.1) is given as:

$$MSE(Y) = \theta^2 \left[na(na-2k)^{-1} \delta^{2k-2} - 2na(na-k)^{-1} \delta^{k-1} + 1 \right] \qquad \text{where } \delta = \theta\theta_0^{-1}.$$

Now we propose the preliminary test shrunken estimator for θ as

$$T_1 = \begin{cases} Y & if\ \theta_0^{-1} \alpha^{\frac{1}{na}} \leq X_{(n)} \leq \theta_0^{-1} \\ \hat{\theta} & ;Otherwise \end{cases} \tag{2.2}$$

The expected value of T_1 is obtained as

$$E(T_1) = \begin{cases} E_1(T_1) & \text{if } \theta^{-1} < \theta_0^{-1} \alpha^{\frac{1}{na}} \\ E_2(T_1) & \text{if } \theta_0^{-1} \alpha^{\frac{1}{na}} \leq \theta^{-1} \leq \theta_0^{-1} \\ E_3(T_1) & \text{if } \theta_0^{-1} < \theta^{-1} \end{cases}$$

where

$$E_1(T_1) = \frac{na\theta}{na-1}$$

$$E_2(T_1) = \frac{na\theta_0}{(na-k)} (\delta)^k \left(1 - (\delta)^{na-k} \alpha^{\frac{na-k}{na}} \right) + \frac{na\theta_0}{(na-1)} (\delta)^{na} \alpha^{\frac{na-1}{na}}$$

$$E_3(T_1) = \frac{na\theta_0}{(na-k)} (\delta)^{na} \left(1 - \alpha^{\frac{na-k}{na}} \right) + \frac{na\theta_0}{(na-1)} (\delta)^{na} \alpha^{\frac{na-1}{na}} + \frac{na\theta}{(na-1)} \left(1 - (\delta)^{na-1} \right)$$

The expression for mean square error of T_1 is obtained as

$$MSE(T_1) = \begin{cases} MSE_1(T_1) & \text{if } \theta^{-1} < \theta_0^{-1} \alpha^{\frac{1}{na}} \\ MSE_2(T_1) & \text{if } \theta_0^{-1} \alpha^{\frac{1}{na}} \leq \theta^{-1} \leq \theta_0^{-1} \\ MSE_3(T_1) & \text{if } \theta_0^{-1} < \theta^{-1} \end{cases}$$

Where

$$MSE_1(T_1) = 2\theta^2 \left\{ (na-1)(na-2) \right\}^{-1} = MSE(\hat{\theta}),$$

$$MSE_2(T_2) = \theta^2\left[\frac{na}{(na-2k)}(\delta)^{2k-2}\left\{1-(\delta)^{na-2k}\alpha^{\frac{na-2k}{na}}\right\}\right.$$

$$+\frac{na}{(na-2)}(\delta)^{na-2}\alpha^{\frac{na-2}{na}} - \frac{2na}{(na-k)}(\delta)^{k-1}\left\{1-(\delta)^{na-k}\alpha^{\frac{na-k}{na}}\right\}$$

$$\left. - \frac{2na}{(na-1)}(\delta)^{na-1}\alpha^{\frac{na-1}{na}} + 1\right],$$

$$MSE_3(T_2) = \theta^2\left[\frac{na}{(na-2k)}(\delta)^{na-2}\left\{1-\alpha^{\frac{na-2k}{na}}\right\} + \frac{na}{(na-2)}(\delta)^{na-2}\alpha^{\frac{na-2}{na}}\right.$$

$$+\frac{na}{(na-2)}\left\{1-(\delta)^{na-2}\right\} - \frac{2na}{(na-k)}(\delta)^{na-1}\left\{1-\alpha^{\frac{na-k}{na}}\right\}$$

$$\left. - \frac{2na}{(na-1)}(\delta)^{na-1}\alpha^{\frac{na-1}{na}} - \frac{2na}{(na-1)}\left\{1-(\delta)^{na-1}\right\} + 1\right].$$

The relative efficiency of the proposed preliminary test shrunken estimators T_i with respect to $\hat{\theta}$ is given by

$$RE(T_1,\hat{\theta}) = \begin{cases} RE_1(T_1,\hat{\theta}) & if\ \theta^{-1} < \theta_0^{-1}\alpha^{\frac{1}{na}} \\ RE_2(T_1,\hat{\theta}) & if\ \theta_0^{-1}\alpha^{\frac{1}{na}} \le \theta^{-1} \le \theta_0^{-1} \\ RE_3(T_1,\hat{\theta}) & if\ \theta_0^{-1} < \theta^{-1} \end{cases} \tag{2.3}$$

where $RE_1(T_1,\hat{\theta}) = \dfrac{MSE(\hat{\theta})}{MSE_1(T_1)} = 1,\ RE_2(T_1,\hat{\theta}) = \dfrac{MSE(\hat{\theta})}{MSE_2(T_1)}$

and $RE_3(T_1,\hat{\theta}) = \dfrac{MSE(\hat{\theta})}{MSE_3(T_1)}$.

The $RE(T_1,\hat{\theta})$ is the function of n(sample Size), δ(ratio of true value of the parameter to its guess value), a(shape parameter), k (shrinkage factor) and α (level of significance). For selected values of n = 03, 05, 08; a = 0.75, 1.0; δ = 0.60(0.20)1.6; k = 0.1(0.2)0.9 and α = 0.01, 0.05, 0,1, the relative efficiency of the proposed preliminary test estimator T_1 has been calculated and presented them respectively for a = 0.75 and 1.0 in Tables 2.1 to 2.2.

From Tables 2.1 and 2.2 it can be observed that the preliminary test shrunken estimator T_1 performs better than $\hat{\theta}$ for small considered values of n, k and α. The relative efficiency decreases as α, and k increases. The efficient range (range of δ) decreases as values of n and α increases. The proposed estimator is more efficient than $\hat{\theta}$ in the whole considered region of δ for small values of k and α. The maximum gain in efficiency is obtained when value of δ is very close or exactly equal to one for large values of n and a. For small n the maximum efficiency doesn't meet at δ =1, but as we increase value of n or value of shape parameter 'a' the maximum gain in efficiency always attains at δ =1. It is also clear from the tables that the range of relative efficiency decreases as value of shape parameter increases. But the gain in efficiency increases as a increases.

When the hypothesis H_0: $\theta = \theta_0$ is accepted against $H_1 : \theta \neq \theta_0$ then

$$\theta_0^{-1}\alpha^{\frac{1}{na}} \leq X_{(n)} \leq \theta_0^{-1}.$$

which implies that

$$0 \leq \left(\frac{X_{(n)} - \theta_0^{-1}\alpha^{\frac{1}{na}}}{\theta_0^{-1} - \theta_0^{-1}\alpha^{\frac{1}{na}}} \right)^2 \leq 1 .$$

Thus, a choice of shrinkage factor k based on the test statistic is given by

$$k^* = \left(\frac{X_{(n)} - \theta_0^{-1}\alpha^{\frac{1}{na}}}{\theta_0^{-1} - \theta_0^{-1}\alpha^{\frac{1}{na}}} \right)^2 .$$

Following Waiker et al. (1984) and Pandey et al. (1988) and others we may propose preliminary test shrinkage estimator T_2 as

$$T_2 = \begin{cases} Y_1 & \text{if } \theta_0^{-1}\alpha^{\frac{1}{na}} \leq X_{(n)} \leq \theta_0^{-1} \\ \hat{\theta} & ;\text{Otherwise} \end{cases}$$

where $Y_1 = \left(\hat{\theta}\right)^{k^*} . \left(\theta_0\right)^{1-k^*}$.

The expression for $MSE(T_2)$ is obtained as

$$MSE(T_2) = \begin{cases} MSE_1(T_2) & \text{if } \theta^{-1} < \theta_0^{-1}\alpha^{\frac{1}{na}} \\ MSE_2(T_2) & \text{if } \theta_0^{-1}\alpha^{\frac{1}{na}} \leq \theta^{-1} \leq \theta_0^{-1} \\ MSE_3(T_2) & \text{if } \theta_0^{-1} < \theta^{-1} \end{cases}$$

where $MSE_1(T_2) = 2\theta^2\left\{(na-1)(na-2)\right\}^{-1} = MSE(\hat{\theta})$,

$$MSE_2(T_2) = \theta_0^2 I_1\left[\left(\frac{\hat{\theta}}{\theta_0}\right)^{2k^*}\right] + I_2\left[\hat{\theta}^2\right] - 2\theta\theta_0 I_1\left[\left(\frac{\hat{\theta}}{\theta_0}\right)^{k^*}\right] - 2\theta I_2\left[\hat{\theta}\right] + \theta^2,$$

$$MSE_3(T_2) = \theta_0^2 I_3\left[\left(\frac{\hat{\theta}}{\theta_0}\right)^{2k^*}\right] + I_2\left[\hat{\theta}^2\right] + I_4\left[\hat{\theta}^2\right] - 2\theta\theta_0 I_3\left[\left(\frac{\hat{\theta}}{\theta_0}\right)^{k^*}\right]$$
$$- 2\theta I_2\left[\hat{\theta}\right] - 2\theta I_4\left[\hat{\theta}\right] + \theta^2,$$

$$I_1[x] = \int_{\theta_0^{-1}\alpha^{\frac{1}{na}}}^{\theta^{-1}} x.f(X_{(n)})\,dX_{(n)}$$

$$I_2[x] = \int_0^{\theta_0^{-1}\alpha^{\frac{1}{na}}} x.f(X_{(n)})\,dX_{(n)} \quad ; \quad I_3[x] = \int_{\theta_0^{-1}\alpha^{\frac{1}{na}}}^{\theta_0^{-1}} x.f(X_{(n)})\,dX_{(n)}$$

and $I_4[x] = \int_{\theta_0^{-1}}^{\theta^{-1}} x.f(X_{(n)})dX_{(n)}$

The relative efficiency $RE(T_2, \hat{\theta})$ of the preliminary test estimator T_2 with respect to $\hat{\theta}$ can be obtained on similar lines as we obtained for T_1 by simply replacing k with k^*.

The calculated values of the $RE(T_2, \hat{\theta})$ are presented in Table 2.3 for selected values of n=03, 05, 08; α= 0.01, 0.05, 0.10; δ = 0.6(0.20)1.6 and a = 0.75, 1.0. It may be observed from the table that the proposed estimator T_2 is more efficient than $\hat{\theta}$ in the whole considered range of δ only for n = 3, and a = 0.75. As we increase n, the efficient range reduces to $0.6 \le \delta \le 1.2$ for small α and becomes $0.6 \le \delta \le 1.0$ for large values α. The maximum efficiency attains at δ =1 only for large n. The rest performance of this estimator is exactly same as we describe for T_4.

3. Conclusion

From the above discussion it is obvious that by using point guess value one can improve the existing estimator. It can be noted that if the point guess value is very close to the true value of the parameter (i.e if δ is approx. close to one), the proposed estimators perform better than existing estimator. If one has no confidence in the guessed value then proposed preliminary test estimators can be suggested. We can safely use the proposed estimators for small sample sizes, at usual level of significance and moderate values of shrunken factor k.

Table 2.1 :: Values of $RE(T_1, \hat{\theta})$ for a = 0.75

n	k	α	δ					
			0.6	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**
3		**0.01**	1.3220	1.2898	1.2475	1.2016	1.1598	1.124
	0.1	**0.05**	1.1512	1.1277	1.0991	1.0691	1.0428	1.0217
		0.1	1.0943	1.0762	1.0547	1.0323	1.0137	1.0000
		0.01	1.2944	1.2709	1.2382	1.2014	1.1665	1.1355
	0.3	**0.05**	1.1353	1.1172	1.0946	1.0703	1.0483	1.0299
		0.1	1.0830	1.0690	1.0519	1.0338	1.0181	1.0061
		0.01	1.2493	1.2348	1.2127	1.1866	1.1606	1.1366
	0.5	**0.05**	1.1117	1.0993	1.0832	1.0654	1.0487	1.0341
		0.1	1.0674	1.0575	1.0452	1.0319	1.0199	1.0102
		0.01	1.1779	1.1715	1.1600	1.1454	1.1302	1.1154
	0.7	**0.05**	1.0777	1.0708	1.0615	1.0508	1.0404	1.0309
		0.1	1.0461	1.0403	1.0330	1.0248	1.0172	1.0108
		0.01	1.0701	1.0690	1.0661	1.0621	1.0577	1.0531
	0.9	**0.05**	1.0300	1.0280	1.0252	1.0217	1.0182	1.0149
		0.1	1.0175	1.0157	1.0133	1.0106	1.0080	1.0056
5		**0.01**	1.9265	2.4964	2.4739	1.8328	1.3490	1.0638
	0.1	**0.05**	1.4824	1.6178	1.4992	1.2101	0.9967	0.8792
		0.1	1.3128	1.3663	1.2658	1.0710	0.9411	0.9011
		0.01	1.7410	2.1930	2.3110	1.9196	1.5098	1.2243
	0.3	**0.05**	1.3918	1.5135	1.4587	1.2494	1.0652	0.9498
		0.1	1.2548	1.3067	1.2458	1.0987	0.9850	0.9395
		0.01	1.5359	1.8417	1.9909	1.8383	1.5826	1.3549
	0.5	**0.05**	1.2897	1.3844	1.3727	1.2472	1.1115	1.0121
		0.1	1.1896	1.2330	1.2032	1.1064	1.0184	0.9735
		0.01	1.3201	1.4820	1.5833	1.5645	1.4713	1.3564
	0.7	**0.05**	1.1782	1.2369	1.2443	1.1893	1.1139	1.0475
		0.1	1.1178	1.1468	1.1372	1.0872	1.0328	0.9978
		0.01	1.1045	1.1491	1.1796	1.1871	1.1769	1.1559
	0.9	**0.05**	1.0603	1.0795	1.0854	1.0744	1.0542	1.0323
		0.1	1.0404	1.0507	1.0500	1.0368	1.0193	1.0051

8		**0.01**	1.3646	2.3192	3.5209	1.5548	0.8117	0.5653
	0.1	**0.05**	1.2492	1.6804	1.7792	1.0347	0.7343	0.8402
		0.1	1.1822	1.4342	1.4060	0.9364	0.8586	1.0000
		0.01	1.2937	1.9866	3.0619	1.8893	1.0582	0.7299
	0.3	**0.05**	1.2002	1.5324	1.6939	1.1522	0.8429	0.8901
		0.1	1.1462	1.3453	1.3685	1.0095	0.9078	1.0000
		0.01	1.2161	1.6641	2.3657	1.9972	1.3251	0.9470
	0.5	**0.05**	1.1471	1.3772	1.5330	1.2207	0.9499	0.9377
		0.1	1.1074	1.2497	1.2941	1.0600	0.9520	1.0000
		0.01	1.1329	1.3699	1.7047	1.7104	1.4253	1.1429
	0.7	**0.05**	1.0905	1.2217	1.3255	1.2014	1.0260	0.9767
		0.1	1.0661	1.1502	1.1896	1.0723	0.9854	1.0000
		0.01	1.0451	1.1133	1.1958	1.2257	1.1919	1.1216
	0.9	**0.05**	1.0308	1.0716	1.1058	1.0851	1.0318	0.9985
		0.1	1.0225	1.0498	1.0656	1.0362	1.0010	1.0000

Table 2.2 :: Values of $RE(T_1, \hat{\theta})$ for a = 1.0

n	k	α	δ					
			0.6	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**
3		**0.01**	1.9393	2.0631	1.9264	1.6443	1.3984	1.2187
	0.1	**0.05**	1.4358	1.4339	1.3344	1.1878	1.0661	0.9819
		0.1	1.2718	1.2557	1.1805	1.0777	0.9981	0.9525
		0.01	1.7809	1.9115	1.8564	1.6651	1.4673	1.3059
	0.3	**0.05**	1.3657	1.3769	1.3124	1.2023	1.1009	1.0247
		0.1	1.2275	1.2215	1.1688	1.0895	1.0223	0.9797
		0.01	1.586	1.6968	1.6973	1.5997	1.4730	1.3536
	0.5	**0.05**	1.2795	1.2966	1.2624	1.1913	1.1176	1.0565
		0.1	1.1741	1.1747	1.1425	1.0883	1.0376	1.0018
		0.01	1.3614	1.4314	1.4514	1.4225	1.3688	1.3079
	0.7	**0.05**	1.1775	1.1927	1.1802	1.1450	1.1033	1.0646
		0.1	1.1112	1.1146	1.0992	1.0695	1.0385	1.0139
		0.01	1.1207	1.1426	1.1528	1.1516	1.1430	1.1304
	0.9	**0.05**	1.0619	1.0682	1.0666	1.0581	1.0463	1.0338
		0.1	1.0392	1.0412	1.0375	1.0290	1.0191	1.0100
5		**0.01**	1.5731	2.5398	3.1407	1.7631	1.0432	0.7419
	0.1	**0.05**	1.3583	1.7135	1.6816	1.1348	0.8349	0.7590
		0.1	1.2505	1.4386	1.3577	1.0049	0.8546	1.0000
		0.01	1.4579	2.160	2.8074	2.0039	1.2902	0.9287
	0.3	**0.05**	1.2874	1.5672	1.6139	1.2218	0.9375	0.8403
		0.1	1.2011	1.3541	1.3268	1.0606	0.9140	1.0000
		0.01	1.3331	1.7810	2.2534	1.9999	1.4989	1.1429
	0.5	**0.05**	1.2105	1.4069	1.4809	1.2559	1.0281	0.9218
		0.1	1.1478	1.2595	1.2642	1.0918	0.9663	1.0000
		0.01	1.2017	1.4326	1.6741	1.6764	1.4898	1.2732
	0.7	**0.05**	1.1287	1.2412	1.3006	1.2105	1.0756	0.9873
		0.1	1.0908	1.1580	1.1728	1.0869	1.0018	1.0000
		0.01	1.0672	1.1312	1.1929	1.2143	1.1952	1.1513
	0.9	**0.05**	1.0435	1.0783	1.1100	1.0846	1.0471	1.0115
		0.1	1.0309	1.0528	1.0608	1.0393	1.0096	1.0000
8		**0.01**	1.1489	1.8616	4.0399	1.1165	0.5173	0.4233
	0.1	**0.05**	1.1115	1.5247	1.9070	0.8353	0.7303	1.0000
		0.1	1.0860	1.3591	1.4685	0.8182	1.0000	1.0000
		0.01	1.1211	1.6549	3.3768	1.5005	0.7061	0.5439
	0.3	**0.05**	1.0900	1.4083	1.7954	0.9814	0.8129	1.0000
		0.1	1.0691	1.2826	1.4213	0.9079	1.0000	1.0000
		0.01	1.0903	1.4513	2.4845	1.8016	0.9679	0.7137
	0.5	**0.05**	1.0666	1.2891	1.5957	1.1033	0.8954	1.0000
		0.1	1.0509	1.2028	1.3311	0.9839	1.0000	1.0000

0.7	**0.01**	1.0564	1.2586	1.7306	1.6923	1.2033	0.9144
	0.05	1.0413	1.1706	1.3534	1.1466	0.9643	1.0000
	0.1	1.0315	1.1216	1.2093	1.0279	1.0000	1.0000
0.9	**0.01**	1.0196	1.0816	1.1969	1.2356	1.1508	1.0302
	0.05	1.0142	1.0555	1.1118	1.0744	1.0004	1.0000
	0.1	1.0108	1.0402	1.0709	1.0231	1.0000	1.0000

Table 2.3 :: Values of $RE(T_2, \hat{\theta})$

a	**n**	**α**	**δ**					
			0.6	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**
a =0.75	**3**	**0.01**	1.3243	1.2985	1.2625	1.2210	1.1792	1.1412
		0.05	1.1592	1.1393	1.1143	1.0865	1.0595	1.0366
		0.10	1.1045	1.0892	1.0702	1.0493	1.0299	1.0149
	5	**0.01**	1.7551	2.2312	2.3755	1.9168	1.3894	1.0572
		0.05	1.4163	1.5456	1.4808	1.2270	0.9905	0.8570
		0.10	1.2758	1.3301	1.2579	1.0762	0.9290	0.8837
	8	**0.01**	1.2816	1.9463	3.1218	1.7548	0.8060	0.5247
		0.05	1.2059	1.5531	1.7281	1.0495	0.6976	0.8157
		0.10	1.1552	1.3689	1.3874	0.9280	0.8343	1.0000
a =1.0	**3**	**0.01**	1.8171	1.9555	1.8943	1.6723	1.4219	1.2224
		0.05	1.3916	1.4005	1.3262	1.1960	1.0667	0.9725
		0.10	1.2468	1.2376	1.1766	1.0813	0.9942	0.9425
	5	**0.01**	1.4476	2.1455	2.8819	1.9407	1.0652	0.7103
		0.05	1.2994	1.5959	1.6445	1.1570	0.8095	0.7248
		0.10	1.2154	1.3802	1.3435	1.0056	0.8296	1.0000
	8	**0.01**	1.1131	1.6063	3.3850	1.2515	0.4818	0.3811
		0.05	1.0912	1.4164	1.8324	0.8225	0.6921	1.0000
		0.10	1.0694	1.2989	1.4430	0.7905	1.0000	1.0000

References

1. Davis, R. L. and Arnold, J. C. (1970). An efficient preliminary test estimator for the variance of normal population when mean is unknown, Biometrika, 56, p. 674- 676

2. Mehta, J.S. and Srinivasan R. (1971). Estimation of mean by shrinkage to a point, Jour. Amer. Statist. Assoc., 66, p. 86-90.

3. Pandey, B. N. (1979 a). On shrinkage estimation of Normal population variance. Communications in Statistics -Theory and Methods, 8, p. 359-365.

4. Pandey, B.N., Malik, H.J. and Srivastava, R. (1988). Shrinkage estimator for the variance of a normal distributions at single and double stages, Microelectron Reliability, 28 (6), p.929-944.

5. Prakash,G., Singh,D.C., and Singh R.D. (2006). Some test estimator for the scale parameter of classical Pareto distribution. Journals of Statistical Research, 40(2), p. 41- 54.

6. Singh, D.C., Prakash, G., and Singh P. (2007). Shrinkage testimators for the shape parameter of Pareto distribution using LINEX loss function. Communication in statistics – Theory and methods, 36(4), p. 741- 753.

7. Singh, D.C., Singh, P. and Singh, P.R. (1996), Shrunken estimators for the scale parameter of Classical Pareto Distribution, Microelectron Reliability, 36 (3), p. 435-439.

8. Singh, H.P., and Shukla, S.K. (2000). Estimation in the two parameter Weibull Distribution with prior Information, IAPQR Transactions, 25(2), p. 107- 118.

9. Saleh, A. K. E. (2006). Theory of Preliminary Test and Stein – type Estimators with Application, Wiley and Sons, New York.

10.Thompson, J. R. (1968 a). Some shrinkage techniques for estimating the mean. Journal of the American Statistical Association, 63, p. 113- 122.