Journal of Reliability and Statistical Studies (ISSN: 0974-8024)
Vol. 1, Issue 1(2008) 33-41

RELIABILITY ANALYSIS OF A SYSTEM OF BOILER USED IN READYMADE GARMENT INDUSTRY

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Abstract
The present paper deals with the reliability analysis of a system of boiler used in garment industry. The system consists of a single unit of boiler which plays an important role in garment industry. Using regenerative point technique with Markov renewal process various reliability characteristics of interest are obtained.

Key Words: Reliability, Markov Process, Regenerative Point Technique.

1. Introduction
In the present paper an attempt has been made to evaluate the reliability of a system of boilers used in readymade garments industry. The boiler is a very important part of a garment industry.

> "A boiler is a closed vessel in which water or the fluid is heated under pressure. The steam or hot fluid is then circulated out of the boiler for use in various process or heating applications. A safety valve is required to prevent over pressurization and possible explosion of a boiler."

The boiler is used for a number of applications in readymade garment industry viz.

(i) heating water for washing machines
(ii) to generate steam for tumblers to dry garments
(iii) to generate steam for steam heated fusing machines
(iv) to generate steam for irons

In view of the above listed applications of a boiler in a garment factory its failure in any case is not tolerable. These boilers may be failed due to a number of reasons, some of the main reasons are as follows:

Failures of mechanical/electrical safety valves

If the pressure in the boiler goes beyond the required limits, these valves are released. There is a led, which will melt and release the pressure.

(a) Failures of temperature sensors

In case of failures, metallic sensors which sense the temperature cut the electrical supply coming to the heater and stop the working of boiler.

(b) Failures due to non-supply of water in the heater

If water is not supplied, air inside the boiler will get heated and will cause the blast of the boiler.

Keeping the above view, we in this paper analyzed a system of boiler in which the policy of preventive maintenance is applied after continuous working for a random amount of time to make the system more reliable. In the system the two repair facilities are considered named as regular and expert repairman. If the regular repairman is able to repair the failed boiler within the fixed amount of time then it is O.K. otherwise expert repairman will be called.

Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.

(i) Transition and steady state transition probabilities
(ii) Mean Sojourn times in various states
(iii) Mean time to system failure (MTSF)

(iv) Point wise and Steady state availability of the system
(v) Expected Busy period of the ordinary repair facility in (0,t]
(vi) Expected Busy period of the expert repair facility in (0,t]
(vii) Expected number of visits by the ordinary repairman in (0,t]
(viii) Expected number of visits by the expert repairman in (0,t]

2. Model Description and Assumptions

(i) The system consists of a single unit of boiler, which is operative initially.
(ii) A system of boiler can fail due to three reasons as
  (a) Failure of mechanical/electrical safety valves
  (b) Failure of temperature sensors
  (c) Failure due to non supply of water in the heaters
(iii) The probabilities that a boiler will fail due to reason (a), (b) and (c) are fixed.
(iv) The policy of preventive maintenance is applied after continuously working for a random amount of time to make the system more reliable. In this the system becomes down (not failed) and the complete unit is inspected and oiling etc. are applied.
(v) There are two types of repair facility in the system known as regular and expert. Whenever, a boiler fails with any of the reason, the failed unit is sent for repair by the regular repair facility. If the regular repairman is able to repair the failed unit within the fixed amount of time known as "Patience time" then it is OK otherwise the expert repairman will be called. Once the expert repairman enters, it will complete all the jobs related to the system. The preventive maintenance will be completed by the regular repair facility only.
(vi) The distribution of time to failure of a working boiler and the time after which policy of preventive maintenance will be apply are exponential with different parameters.
(vii) The distribution of completing preventive maintenance, repair of failed boiler by both the repair facility are general.

3. Notations and Symbols

N_O	:	Normal unit of boiler under operation
N_{pm}	:	Normal unit of boiler under preventive maintenance
$F^{(a)}_{Or}$	:	Boiler under ordinary repairman failed due to reason (a)
$F^{(b)}_{Or}$	:	Boiler under ordinary repairman failed due to reason (b)
$F^{(c)}_{Or}$	:	Boiler under ordinary repairman failed due to reason (c)
F_{er}	:	Failed boiler under expert repairman
α	:	Constant failure rate of an operative boiler
β	:	Constant rate of applying the policy of preventive maintenance
$g(.), G(.)$	:	pdf and cdf of time to complete preventive maintenance
$h_1(.), H_1(.)$	:	pdf and cdf of time to complete repair of a boiler failed due to reason (a)
$h_2(.), H_2(.)$	:	pdf and cdf of time to complete repair of a boiler failed due to reason (b)
$h_3(.), H_3(.)$	:	pdf and cdf of time to complete repair of a boiler failed due to reason (c)
$f(.), F(.)$	:	pdf and cdf of time to completing patience time for ordinary repairman
$k(.), K(.)$	:	pdf and cdf of time to complete repair of a boiler by expert repairman
p	:	Probability that an operative boiler will fail due to reason (a)
q	:	Probability that an operative boiler will fail due to reason (b)
r	:	Probability that an operative boiler will fail due to reason (c)
m_1	:	Mean patience time = $\int_0^\infty t.g(t)\,dt$
m_2	:	Mean repair time of a boiler by expert repairman = $\int_0^\infty t.k(t)\,dt$

Using the above notation and symbols the possible states of the system are

Up States : $S_0 \equiv (N_O)$
Down States : $S_1 \equiv (N_{pm})$
Failed States : $S_2 \equiv (F^{(a)}_{Or})$ $S_3 \equiv (F^{(b)}_{Or})$ $S_4 \equiv (F^{(c)}_{Or})$ $S_5 \equiv (F_{er})$

The transitions between the various states are shown in Fig. 1.

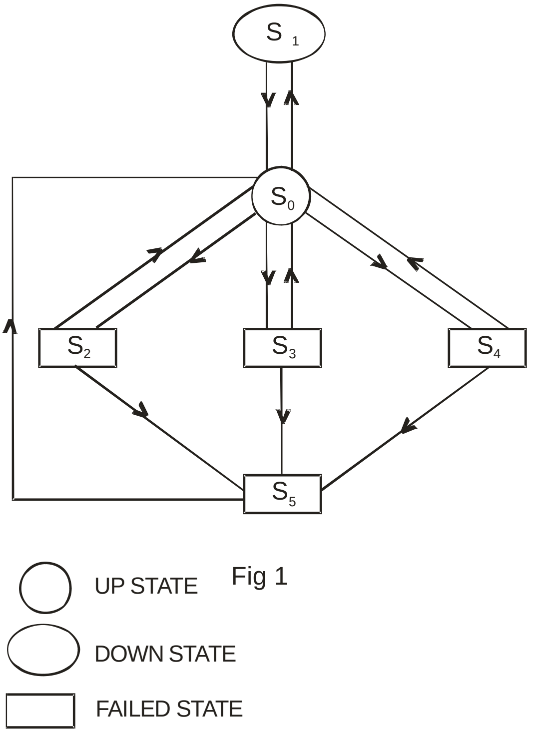

Fig 1

4. Transition Probabilities

Let T_0 (=0), $T_1, T_2, \ldots$ be the epochs at which the system enters the states $S_i \in E$. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = Pr[X_{n+1} = S_k, T_{n+1} - T_n \leq t \mid X_n = S_i] \quad (1)$$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty) \quad (2)$$

By simple probabilistic consideration, the non-zero elements of $Q_{ik}(t)$ are:

$$Q_{01}(t) = \int_0^t \beta e^{-(\alpha+\beta)u}\, du \qquad Q_{02}(t) = \int_0^t p.\alpha e^{-(\alpha+\beta)u}\, du$$
$$Q_{03}(t) = \int_0^t q.\alpha e^{-(\alpha+\beta)u}\, du \qquad Q_{04}(t) = \int_0^t r.\alpha e^{-(\alpha+\beta)u}\, du$$
$$Q_{10}(t) = \int_0^t g(u)\, du \qquad Q_{20}(t) = \int_0^t \overline{F}(u)\, h_1(u)\, du$$
$$Q_{25}(t) = \int_0^t f(u)\, \overline{H}_1(u)\, du \qquad Q_{30}(t) = \int_0^t \overline{F}(u)\, h_2(u)\, du$$
$$Q_{35}(t) = \int_0^t f(u)\, \overline{H}_2(u)\, du \qquad Q_{40}(t) = \int_0^t \overline{F}(u)\, h_3(u)\, du$$
$$Q_{45}(t) = \int_0^t f(u)\, \overline{H}_3(u)\, du \qquad Q_{50}(t) = \int_0^t k(u)\, du \quad (3)$$

Taking limit as $t\to\infty$, the steady state transition probabilities p_{ij}'s can be obtained from (3) . Thus

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) \quad (4)$$

$$p_{01} = \frac{\beta}{\alpha+\beta} \qquad p_{02} = p.\frac{\alpha}{\alpha+\beta} \qquad p_{03} = q.\frac{\alpha}{\alpha+\beta}$$
$$p_{04} = r.\frac{\alpha}{\alpha+\beta} \qquad p_{10} = 1 \qquad p_{20} = \int_0^\infty \overline{F}(t)\, h_1(t)\, dt$$
$$p_{25} = \int_0^\infty f(t)\, \overline{H}_1(t)\, dt \qquad p_{30} = \int_0^\infty \overline{F}(t)\, h_2(t)\, dt$$
$$p_{35} = \int_0^\infty f(t)\, \overline{H}_2(t)\, dt \qquad p_{40} = \int_0^\infty \overline{F}(t)\, h_3(t)\, dt$$
$$p_{45} = \int_0^\infty f(t)\, \overline{H}_3(t)\, dt \qquad p_{50} = 1 \quad (5)$$

From the above probabilities the following relations can be easily verified as;

$$p_{01} + p_{02} + p_{03} + p_{04} = 1 \qquad p_{10} = 1 = p_{50}$$
$$p_{20} + p_{25} = 1 \qquad p_{30} + p_{35} = 1$$
$$p_{40} + p_{45} = 1 \quad (6)$$

5. Mean Sojourn Times

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = \int_0^\infty P[T>t]\, dt \quad (7)$$

Where T is the time of stay in state S_i by the system.

To calculate mean sojourn time μ_I in state S_i, we assume that so long as the system is in state S_i, it will not transit to any other state. Therefore;

$$\mu_0 = \int_0^\infty e^{-(\alpha+\beta)t}\, dt = 1/(\alpha+\beta)$$
$$\mu_1 = \int_0^\infty \overline{G}(t)\, dt = \int_0^\infty t.g(t)\, dt = m_1$$
$$\mu_2 = \int_0^\infty \overline{F}(t)\, \overline{H}_1(t)\, dt$$
$$\mu_3 = \int_0^\infty \overline{F}(t)\, \overline{H}_2(t)\, dt$$

$\mu_4 = {}_0\int^\infty \overline{F}(t)\,\overline{H}_3(t)\,dt$

$\mu_5 = {}_0\int^\infty \overline{K}(t)\,dt = {}_0\int^\infty t.k(t)\,dt = m_2$ (8)

Contribution to Mean Sojourn Time

For the contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state occurs, before transiting to $S_j \in E$, i.e.,

$m_{ij} = {}_0\int^\infty t.q_{ij}(t)\,dt = -q'^{*}_{ij}(0)$ (9)

Therefore,

$m_{01} = {}_0\int^\infty \beta.t.e^{-(\alpha+\beta)t}\,dt$ $m_{02} = {}_0\int^\infty p.\alpha.t.e^{-(\alpha+\beta)t}\,dt$

$m_{03} = {}_0\int^\infty q.\alpha.t.e^{-(\alpha+\beta)t}\,dt$ $m_{04} = {}_0\int^\infty r.\alpha.t.e^{-(\alpha+\beta)t}\,dt$

$m_{10} = {}_0\int^\infty t.g(t)\,dt = m_1$ $m_{20} = {}_0\int^\infty t.\overline{F}(t)\,h_1(t)\,dt$

$m_{25} = {}_0\int^\infty t.f(t)\,\overline{H}_1(t)\,dt$ $m_{30} = {}_0\int^\infty t.\overline{F}(t)\,h_2(t)\,dt$

$m_{35} = {}_0\int^\infty t.f(t)\,\overline{H}_2(t)\,dt$ $m_{40} = {}_0\int^\infty t.\overline{F}(t)\,h_3(t)\,dt$

$m_{45} = {}_0\int^\infty t.f(t)\,\overline{H}_3(t)\,dt$ $m_{50} = {}_0\int^\infty t.k(t)\,dt = m_2$

(10)

By the above expressions, it can be easily verified that

$m_{01} + m_{02} + m_{03} + m_{04} = \mu_0$ $m_{10} = m_1 = \mu_1$

$m_{20} + m_{25} = \mu_2$ $m_{30} + m_{35} = \mu_3$

$m_{40} + m_{45} = \mu_4$ $m_{50} = m_2 = \mu_5$

(11)

6. Mean Time to System Failure (MTSF)

To obtain the distribution function $\pi_i(t)$ of the time to system failure with starting state S_0.

$\pi_0(t) = Q_{01}(t)\$\pi_1(t) + Q_{02}(t) + Q_{03}(t) + Q_{04}(t)$

$\pi_1(t) = Q_{10}(t)\$\pi_0(t)$ (12)

Taking Laplace Stieltjes transform of relations (12) and solving for $\tilde{\pi}_0(s)$ by omitting the argument 's' for brevity, we get

$$\tilde{\pi}_0(s) = \frac{N_1'(s)}{D_1(s)} \quad (13)$$

where $N_1(s) = \tilde{Q}_{02} + \tilde{Q}_{03} + \tilde{Q}_{04}$ (14)

and $D_1(s) = 1 - \tilde{Q}_{01}.\tilde{Q}_{10}$ (15)

By taking the limit $s \rightarrow 0$ in equation (13), one gets $\tilde{\pi}_0(s) = 1$, which implies that $\tilde{\pi}_0(s)$ is a proper distribution function. Therefore, mean time to system failure when the initial state is S_0, is

$$E(T) = -\left[\frac{d}{ds}\tilde{\pi}_0(s)\right]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} = \frac{N_1'}{D_1} \quad (16)$$

where $N_1 = \mu_0 + m_1 p_{01}$ (17)

and

$D_1 = 1 - p_{01}$ (18)

7. Availability Analysis

System availability is defined as

$A_i(t)$ = Pr[Starting from state S_i the system is available at epoch t without passing through any regenerative state] and

$M_i(t)$ = Pr[Starting from up state S_i the system remains up till epoch t without passing through any regenerative up state]
Thus,

$$M_0(t) = e^{-(\alpha+\beta)t} \qquad M_1(t) = \bar{G}(t) \qquad (19)$$

Now, obtaining $A_i(t)$ by using elementary probability argument;

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t) + q_{03}(t)©A_3(t) + q_{04}(t)©A_4(t)$
$A_1(t) = M_1(t) + q_{10}(t)©A_0(t)$
$A_2(t) = q_{20}(t)©A_0(t) + q_{25}(t)©A_5(t)$
$A_3(t) = q_{30}(t)©A_0(t) + q_{35}(t)©A_5(t)$
$A_4(t) = q_{40}(t)©A_0(t) + q_{45}(t)©A_5(t)$
$A_5(t) = q_{50}(t)©A_0(t)$ (20)

Taking Laplace transforms of above equations (20) and solving for $A_0^*(s)$, by omitting the argument 's' for brevity, one gets

$$A_0^*(s) = \frac{N_2'(s)}{D_2(s)} \qquad (21)$$

Where

$$N_2(s) = M^*_0 + q^*_{01}M^*_1 \qquad (22)$$

and

$$D_2(s) = 1 - q^*_{01}q^*_{10} - q^*_{02}(q^*_{20} + q^*_{25}q^*_{50}) - q^*_{03}(q^*_{30} + q^*_{35}q^*_{50}) - q^*_{04}(q^*_{40} + q^*_{45}q^*_{50}) \qquad (23)$$

By taking the limit $s\to0$ in the relation (23), one gets the value of $D_2(0) = 0$.

Also, $M^*_0(0) = \mu_0 \qquad M^*_1(0) = m_1$ (24)

Therefore, the steady state availability of the system when it starts operations from S_0 is

$$A_0(\infty) = \lim_{t\to\infty} A_0(t) = \lim_{s\to0} s.A_0^*(s) = N_2(0)/D'_2(0) = N_2/D_2 \qquad (25)$$

where, N_2 and D_2 are as $N_2 = N_2(0) = \mu_0 + p_{01}m_1$ (26)

and $D_2 = D'_2(0) = \mu_0 + m_1(p_{01} + p_{04}p_{45}) + \mu_2p_{02} + \mu_3p_{03} + \mu_4p_{04}$ (27)

8. Busy Period Analysis of Ordinary Repair Facility

Let us define $W_i(t)$ as the probability that the system is under repair by ordinary repair facility in state $S_i \in E$ at time t without transiting to any regenerative state. Therefore

$$W_1(t) = \bar{G}(t),\ W_2(t) = \bar{H}_1(t),\ W_3(t) = \bar{H}_2(t),\ W_4(t) = \bar{H}_3(t) \qquad (28)$$

Also let $B_i(t)$ is the probability that the system is under repair by ordinary repair facility at time t, Thus the following recursive relations among $B_i(t)$'s can be obtained as ;

$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t) + q_{03}(t)©B_3(t) + q_{04}(t)©B_4(t)$
$B_1(t) = W_1(t) + q_{10}(t)©B_0(t)$
$B_2(t) = W_2(t) + q_{20}(t)©B_0(t) + q_{25}(t)©B_5(t)$
$B_3(t) = W_3(t) + q_{30}(t)©B_0(t) + q_{35}(t)©B_5(t)$
$B_4(t) = W_4(t) + q_{40}(t)©B_0(t) + q_{45}(t)©B_5(t)$
$B_5(t) = q_{50}(t)©B_0(t)$ (29)

Taking Laplace transform of the equations (29) and solving for $B^*_0(s)$, by omitting the argument 's' for brevity we get;

$$B_0^*(s) = \frac{N_3^{'}(s)}{D_3(s)} \tag{30}$$

where $D_3(s)$ is same as $D_2(s)$ in (23) and

$$N_3(s) = q^*_{01}W^*_1 + q^*_{02}W^*_2 + q^*_{03}W^*_3 + q^*_{04}W^*_4 \tag{31}$$

In this steady state, the fraction of time for which the ordinary repair facility is busy in repair is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s.B_0^*(s) = N_3(0)/D_3^{'}(0) = N_3/D_3 \tag{32}$$

where D_3 is same as D_2 in (27) and in terms of

$W^*_1(0) = {}_0\int^\infty t.g(t)\,dt = w_1$ (say)

$W^*_2(0) = {}_0\int^\infty t.h_1(t)\,dt = w_2$ (say)

$W^*_3(0) = {}_0\int^\infty t.h_2(t)\,dt = w_3$ (say)

$W^*_4(0) = {}_0\int^\infty t.h_3(t)\,dt = w_4$ (say) (33)

We have

$N_3 = p_{01}w_1 + p_{02}w_2 + p_{03}w_3 + p_{04}w_4$ (34)

9. Busy Period Analysis of Expert Repair Facility

Let us define $W_i(t)$ as the probability that the system is under repair by expert repair facility in state S_i ε E at time t without transiting to any regenerative state. Therefore

$$W_5(t) = \bar{K}(t) \tag{35}$$

Also let $E_i(t)$ is the probability that the system is under repair by expert repair facility at time t, Thus the following recursive relations among $B_i(t)$'s can be obtained as ;

$E_0(t) = q_{01}(t)©E_1(t) + q_{02}(t)©E_2(t) + q_{03}(t)©E_3(t) + q_{04}(t)©E_4(t)$

$E_1(t) = q_{10}(t)©E_0(t)$

$E_2(t) = q_{20}(t)©E_0(t) + q_{25}(t)©E_5(t)$

$E_3(t) = q_{30}(t)©E_0(t) + q_{35}(t)©E_5(t)$

$E_4(t) = q_{40}(t)©E_0(t) + q_{45}(t)©E_5(t)$

$E_5(t) = W_5(t) + q_{50}(t)©E_0(t)$ (36)

Taking Laplace transform of the equations (36) and solving for $E_0^*(s)$, by omitting the argument 's' for brevity we get;

$$E_0^*(s) = \frac{N_4^{'}(s)}{D_4(s)} \tag{37}$$

Where $D_4(s)$ is same as $D_2(s)$ in (23) and

$$N_4(s) = (q^*_{02}q^*_{25} + q^*_{03}q^*_{35} + q^*_{04}q^*_{45})W^*_5 \tag{38}$$

In this steady state, the fraction of time for which the expert repair facility is busy in repair is given by

$$E_0 = \lim_{t\to\infty} E_0(t) = \lim_{s\to 0} s.E_0^*(s) = N_4(0)/D_4^{'}(0) = N_4/D_4 \tag{39}$$

where D_4 is same as D_2 in (27) and in terms of

$$W^*_5(0) = {}_0\int^\infty t.k(t)\,dt = w_5 \text{ (say)} \tag{40}$$

We have

$$N_4 = (p_{02}p_{25} + p_{03}p_{35} + p_{04}p_{45})w_5 \tag{41}$$

10. Expected Number of Visits by the Ordinary Repair Facility

Let we define, $V_i(t)$ as the expected number of visits by the ordinary repair facility in (0,t] given that the system initially started from regenerative state S_i at t=0. Then following recurrence relations among $V_i(t)$'s can be obtained as;

$$V_0(t) = Q_{01}(t)\$[1 + V_1(t)] + Q_{02}(t)\$[1 + V_2(t)] + Q_{03}(t)\$[1 + V_3(t)] + Q_{04}(t)\$[1 + V_4(t)]$$

$V_1(t) = Q_{10}(t)\$V_0(t)$

$V_2(t) = Q_{20}(t)\$V_0(t) + Q_{25}(t)\$V_5(t)$

$V_3(t) = Q_{30}(t)\$V_0(t) + Q_{35}(t)\$V_5(t)$

$V_4(t) = Q_{40}(t)\$V_0(t) + Q_{45}(t)\$V_5(t)$

$V_5(t) = Q_{50}(t)\$V_0(t)$ (42)

Taking Laplace stieltjes transform of the equations (42) and solving for $V_0^{**}(s)$ by omitting the argument 's' for brevity is

$$V_0^{**}(s) = \frac{N_5^{'}(s)}{D_5(s)} \tag{43}$$

where

$$N_5(s) = \tilde{Q}_{01} + \tilde{Q}_{02} + \tilde{Q}_{03} + \tilde{Q}_{04} \tag{44}$$

and

$$D_5(s) = 1 - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{02}(\tilde{Q}_{20} + \tilde{Q}_{25}\tilde{Q}_{50}) - \tilde{Q}_{03}(\tilde{Q}_{30} + \tilde{Q}_{35}\tilde{Q}_{50}) - \tilde{Q}_{04}(\tilde{Q}_{40} + \tilde{Q}_{45}\tilde{Q}_{50}) \tag{45}$$

In steady state the number of visits per unit of time when the system starts after entrance into state S_0 is

$$V_0 = \lim_{t\to\infty}(V_0(t)/t) = \lim_{s\to 0} s.V_0^{**}(s) = N_5 / D_5 \tag{46}$$

where D_5 is same as D_2 in (27) and

$N_4 = p_{01} + p_{02} + p_{03} + p_{04} = 1$ (47)

11. Expected Number of Visits by the Expert Repair Facility

Let we define, $V_i(t)$ as the expected number of visits by the expert repair facility in (0,t] given that the system initially started from regenerative state S_i at t=0. Then following recurrence relations among $V_i(t)$'s can be obtained as;

$V_0(t) = Q_{01}(t)\$V_1(t) + Q_{02}(t)\$V_2(t) + Q_{03}(t)\$V_3(t) + Q_{04}(t)\$V_4(t)$

$V_1(t) = Q_{10}(t)\$V_0(t)$

$V_2(t) = Q_{20}(t)\$V_0(t) + Q_{25}(t)\$[1 + V_5(t)]$

$V_3(t) = Q_{30}(t)\$V_0(t) + Q_{35}(t)\$[1 + V_5(t)]$

$V_4(t) = Q_{40}(t)\$V_0(t) + Q_{45}(t)\$[1 + V_5(t)]$

$V_5(t) = Q_{50}(t)\$V_0(t)$ (48)

Taking Laplace stieltjes transform of the above equations and solving the equations (48) for $V_0^{**}(s)$ by omitting the argument 's' for brevity is

$$V_0^{**}(s) = \frac{N_6^{'}(s)}{D_6(s)} \tag{49}$$

where

$$N_6(s) = \tilde{Q}_{02}\tilde{Q}_{25} + \tilde{Q}_{03}\tilde{Q}_{35} + \tilde{Q}_{04}\tilde{Q}_{45} \tag{50}$$

and $D_6(s)$ is same $D_5(s)$ as in (45).

In steady state the number of visits per unit of time when the system starts after entrance into state S_0 is ;

$$V_0 = \lim_{t\to\infty}(V_0(t)/t) = \lim_{s\to 0} s.V_0^{**}(s) = N_6 / D_6 \tag{51}$$

where D_6 is same as D_2 in (27) and

$N_6 = p_{02}p_{25} + p_{03}p_{35} + p_{04}p_{45}$ (52)

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