Journal of Reliability and Statistical Studies (ISSN: 0974-8024)
Vol. 1, Issue 1(2008) 61-66

A PROBABILISTIC INVENTORY MODEL FOR CONDITIONAL CREDIT PERIOD AND LEAD TIME WITH MULTIPLE STORAGE FACILITIES

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Abstract

A probabilistic inventory model for conditional credit period with exponential demand, non-zero lead time and multiple storage facility has been developed. The behaviour of total expected cost (TEC) has been examined and the use and application of the model is demonstrated with the help of a numerical example.

Key words: Inventory Model, Conditional Credit Period, Storage Facilities. Total Expected Cost (TEC), Economic Order Quantity (EOQ).

1. Introduction

Goyal [3] and Shah et al. [13] have studied economic order quantity (EOQ) inventory models under the assumptions that buyers agree to pay the supplier at the end of some fixed credit period. After which higher interests are charged if the debt is not settled. This conditional credit period facility is economically advantageous to buyers as it allows them to earn interest from the revenue obtained from their own sales during the credit period. Their model, however, assumed unlimited storage capacity. For an extensive review, one may refer to previous works (for example, [1], [2], [4], [5], [6], [7], [8], [9], [10], [11], [12], [14], [15]and [16]). None of these however considered lead-time to be existent except zero. Recently, Shah and Sreehari [14] have discussed inventory models with multiple storage facilities and conditional credit facility-assuming zero lead-time and period dependent but deterministic demand.

In the present paper, we develop a probabilistic inventory model for conditional credit period with exponential demand, non-zero lead-time and multiple storage facilities assuming that the firm's own warehouse capacity is limited. We obtain an expression for the expected total cost and examine its behaviour, establishing various properties; in particular convexity, a new result for this kind of model. Lastly, we propose an algorithm for the determination of the EOQ and illustrate its use through a numerical example.

2. Assumptions and Notations

We make the following assumptions:

(i) The random demand for the product during the time cycle has Poisson distribution with mean $1/\theta_1$.

(ii) The demand in the lead time L has a Poisson distribution with mean $1/\theta_2$

(iii) Shortages are not allowed, i.e. there is no backlogging.

(iv) The supplier complete payment must be put forward before a subsequent order is made.

(v) Earns simple interest on the income along the initial credit period. The buyer earns interest only from his sales of goods along the credit period.

(vi) The debt at the end of credit period is paid by buyer with interest as and when he sells his whole product.

We use the following notations:

$\Delta = (1/\theta_1) + (1/\theta_2)$; Rate of demand

k = Number of warehouses.

T= Time interval between two consecutive orders, which we can assume to be 1 (change the time unit, if necessary)

t = Credit period (L < t < 1)

c = Unit purchase cost

I_e = Rate at which interest is earned

I_p = Interest charged by the supplier beyond the credit period.
C_0 = Ordering cost (per order)
H_1 = Holding cost at the firm's own warehouse
H_i = Holding cost at alternative warehouse i, $2 \leq i \leq k$ where $H_1 < H_2 < H_3 \ldots < H_k$
W_i = Capacity of ith warehouse i ($1 \leq i \leq k$), where we assume $W_k = \infty$

The assumption $H_1 < H_2 < H_3 \ldots < H_k$ entails no loss of generality as the warehouses can be rearranged and in case of equality the corresponding capacities can be added together. Although realistically there may be other costs such as transportation from the warehouses to the sellers, we will ignore them. Observe that transportation costs can be thought as built into the warehouses' holding costs.

3. The expected cost function

The buyer's cost consists of set up and holding costs minus interest earned during the credit period plus the interest paid on existing debt if any, after the credit period. We observe that initially the buyer gets for min (q, $t\Delta$), if $q > t\Delta$, the buyer would be making a partial payment at the end of the credit period worth $t\Delta$ units period and the remaining amount is paid with interest as and when the surplus inventory is sold.

Let, $V_s = \sum_{i=1}^{s} W_i$ denote the combined capacity of the first s warehouses with $W_0 = V_0 = 0$ and m denote the positive integer such that m<k for which $V_{m-1} < q < V_m$, , i.e. the number of warehouses needed to store q.

Case (i) $q < t\Delta$, i.e. the stock is depleted within the credit period the various components of the total cost per cycle are given below.

Ordering cost = C_0.

Since the available stocks are used for economic reasons, in the decreasing order of holding costs, the amount payable to warehouse j, j < m, will consist of the cost of holding W_j items for a period of $(W_{j+1} + W_{j+2} + \ldots + W_{m-1} + q - V_{m-1})/\Delta$ units of time and on average of $W_j/2$ items for a period of W_j/Δ units of time.

So the corresponding cost is

$$[(W_j^2/\Delta) + \sum_{i=j+1}^{m-1} (W_l + q - V_{m-1})\ W_j/\Delta\ ]\ H_j = W_j H_j (W_j/_2 + q - V_j)/\ \Delta \quad : \text{for } j < m$$

$$= W_m^{\ 2} H_m / \Delta, \quad q = V_m \ : \text{for } j = m$$

(Above and elsewhere, for notational convenience any empty sum is taken to be zero.)

Expected Interest earned per cycle $= cI_e\, [\sigma_0^{\ 2} + \{E(q)/b\}^{\ 2} \{\sigma_1^{\ 2} + b^2\}]\ /2\Delta$

$= cI_e\, [\sigma_0^{\ 2} + (q+q')^2 (\sigma_1^{\ 2} + b^2)]\ /2\Delta = R+S+T$

where, $R = cI_e\, (\sigma_1^{\ 2} + b^2)\, q^2/2\Delta$

$S = \{ cI_e\, q'\, (\sigma_1^{\ 2} + b^2)\} q/\Delta$

$T = (\sigma_0^{\ 2} + q'^2)\, (cI_e)\, (\sigma_1^{\ 2} + b^2)/2\Delta$

and $q' = Q - q$

Q = maximum ordered quantity which can be fulfilled by a seller

b = a real positive quantity

$\sigma_0^{\ 2}$, $\sigma^{\ \ 2}$ have their usual meanings

Total expected cost per unit time, then simplifies to $TEC_1(q) = A_m/q + B_m + C_m q$

where

$$A_m = [2\Delta\, C_0 + V^2_{m-1} H_m + \sum_{j=1}^{m-1} W^2_j H_j - 2 \sum_{j=1}^{m-1} W_j H_j V_j + \quad 2\Delta\, T\ ]/\ 2$$

m-1

$B_m = \sum_{j=1} W_j(H_j - H_m) + S\Delta$

$C_m = (H_m + 2R\Delta) / 2$

It is easily seen that $A_m > 0$ and $B_m < 0$.

Case (ii) $q > t\Delta$

In this case holding cost and ordering cost per cycle are same as in case (i). Other relevant components of cost function are given below.

Expected Interest earned per cycle = $cI_e t^2 \Delta / 2$

Expected Interest paid per cycle = X + Y + Z

where, $X = cI_p(\sigma_1^2+b^2)q^2$

$Y = \{2q'cI_p(\sigma_1^2+b^2)-2tb/\Delta\} q/\Delta$

$Z = cI_p \{(\sigma_0^2+b^2)(\sigma_1^2+b^2) + \Delta^2 t^2 - 2tb\Delta q'\}/\Delta$

Hence, total expected cost per unit time simplifies to

$$TEC_2(q)\ A^*_m/q+B_m^*+C_m^*q$$

where,

$$A_m^* = [2C_0\Delta + V^2_{m-1}H_m + \sum_{j=1}^{m-1} W^2_j H_j - 2\sum_{j=1}^{m-1} W_jH_jV_j + [2\Delta Z - cI_e t^2\Delta^2]/2$$

$$B_m^* = \sum_{j=1}^{m-1} W_jH_j + \Delta Y - V_{m-1}H_m$$

and $C_m^* = (H_m + 2\Delta X)/2$

It is easily seen that

$$A^*_m = A_m + [2(Z-T)\Delta - cI_e t^2\Delta^2]/2$$
$$B^*_m = B_m + (Y-S)\Delta \qquad (1)$$
$$C^*_m = C_m + (X-R)\Delta$$

Also, $A^*_m > 0$, $B^*_m < 0$ and $C^*_m > 0$

Thus, the total expected cost per unit time is

$$TEC(q) = \begin{cases} A_m/q+B_m+C_mq; & \text{if } q \le t\Delta \\ A^*_m/q+B_m^*+C_m^*q; & \text{if } q > t\Delta \end{cases} \qquad (2)$$

We shall now study the behavior of TEC(q).

3.1 Behaviour of TEC (q)

We recall that m, (m < k) is the positive integer such that $V_{m-1} < q \le V_m$. As such m is a function of q. However, for all $q \varepsilon (V_{m-1}, V_m]$ the expressions A_m, B_m, C_m, A_m^*, B_m^* and C_m^* remain constant (free from the exact value of q). We shall prove that TEC (q) is a continuous function of q in $(0,\infty)$ and differentiable in each interval $(V_{m-1}, V_m]$. Further, by the very nature of the functions TEC_1 (q) and TEC_2 (q) in each open interval (V_{m-1}, V_m) they will have only one possible solution for $d/dq\, TEC_i(q) = 0$, i = 1,2.

Although the solution may or may not belong to the interval of definition viz; (V_{m-1}, V_m). Clearly, if the solution is outside this interval it means the functions are monotone throughout the interval. We shall now give details of the behaviour of TEC (q) in the following.

3.2 Proposition 1: The function TEC(q) is continuous on $(0, \infty)$

Proof: It is sufficient to the continuity of the function at the points V_1, V_2. … V_{k-1} and at $t\Delta$ suppose $V_{n-1} < t\Delta < V_n$. Then we have to consider the behaviour of TEC_1 (q) at the points V_1, V_2,……, V_{n-1} and TEC2 (q) at the points V_n, V_{n+1}, …….,V_{k-1} besides proving $TEC_1(t\Delta) = TEC_2(t\Delta)$. If, however, $t\Delta = V_n$, we need to show that $TEC_1(V_n) = TEC_2(V_n+)$. Recall that TEC_1 and TEC_2 are left continuous at all points. The continuity of TEC_1 (q) at, say, $V_r (r \le n-1)$ follows from the fact that $(A_r/V_r) + B_r + C_r V_r = (A_{r+1}/V_r) + B_{r+1} + C_{r+1} V_r$, which is easily verified. Similarly, the continuity of TEC_2 (q) at, say, V_r ($r \geq n$) follows from the fact that $(A_r^*/V_r)+B_r^*+C_r^*V_r^* = (A^*_{r+1}/V_r) + B^*_{r+1} + C^*_{r+1} V_r$, further TEC1 ($t\Delta$) = $A_n/t\Delta + B_n + C_n t\Delta$ and TEC_2 ($t\Delta$) = $A^*_n/t\Delta + B^*_n + C^*_n t\Delta$ and it is fairly easy to observe that they are equal. Finally, if $t\Delta = V_n$ then $TEC_2 (V_n+)= A^*_{n+1}/t\Delta + B^*_{n+1} + C^*_{n+1} t\Delta$. It is easily shown that $TEC_2 (V_n+) = TEC_1 (V_n)$. Thus TEC (q) is continuous on $(0, \infty)$.

3.3 Proposition 2: TEC(q) is differentiable in (0, ∞) except at (t∆).

Proof: Suppose $V_{n-1} < t\Delta < V_n$. It is sufficient to prove that TEC (q) is differentiable at all V_r. Suppose $r \leq n$. Then TEC (q) = TEC_1 (q). The left hand and right hand derivatives of TEC_1 (q) at V_r are respectively $-A_r/(V_r^2) + C_r$ and $-A_{r+1}/(V_r^2) + C_{r+1.}$

Now,

$$\begin{aligned} &C_{r+1} - C_r - (A_{r+1}-A_r)/V_r^2 \\ &= (H_{r+1}-H_r)/2-[V_r^2H_{r+1}-V^2_{r-1}H_r+W_r^2H_r-2W_rH_rV_r]/2V_r^2 \\ &= \{(V^2_{r-1}-V^2_r) Hr-W^2_rH_r+2W_rH_rV_r\}/2V_r^2 \\ &= [\{-W_r(2V_{r-1}+W_r)-W_r^2+2W_rV_r\}H_r]/2V_r^2 \\ &= \{(-2V_{r-1}-2W_r+2V_r) WrHr\}/2V_r^2 \\ &= 0 \end{aligned} \qquad (3)$$

It proves that TEC (q) is differentiable at all $V_r \leq V_{n-1}$. In case $V_r \geq V_{n-1}$; TEC (q) = TEC_2 (q) and the left hand and right hand derivatives of TEC_2 (q) at V_r are $-A_r^*/V_r^2+C_r^*$ and $-A^*_{r+1}/V_r^2+C^*_{r+1}$ respectively. In view of relation (1) and aforesaid results concerning A_r, A_{r+1}, C_r, C_{r+1}, it follows that TEC_2 (q) is differentiable at $V_r \geq V_n$.

Finally, if $t\Delta = V_n$ for some n, then the above steps for $r < n$ and $r > n$, prove differentiability at all points on $(0, \infty)$ except at $t\Delta$.

3.4 Proposition 3: The left hand derivative of TEC(q) at t∆ is less than the right hand derivative of TEC(q) at t∆ and they respectively are $-A_n/(t\Delta)^2 + C_n$ and $-A^*_n/(t\Delta)^2 + C^*_n$.

Proof: In view of relation (1),

$(A_n-A_n^*)/(t\Delta)^2+C_n^*-C_n = [(T-Z)\ \Delta + cI_et^2\Delta^2/2]/(t\Delta)^2+(X-R)\ \Delta$

$[(A_n-A_n^*)/(t\Delta)^2+C_n^*-C_n] > 0$, as claimed.

Next, suppose $t\Delta = V_n$. Then the right hand derivative at V_n is $-A_n^*/V_n^2+C_{n+1}^*$ while the left hand derivative at V_n is $-A_n/V_n^2+C_n$

using (1) and (3) we have $(A_n-A_{n+1}^*)/V_n^2 + C_{n+1}^*-C_n$

$$\begin{aligned} &= (A_n-A_{n+1})/V_n^2+C_{n+1}-C_n +[\{T-Z+cI_et^2\Delta/2\}/t +(X-R)\ \Delta] >0 \\ &= [\{T-Z+cI_et^2\Delta/2\}/t +(X-R)\ \Delta] >0 \end{aligned}$$

This completes the proof of proposition 3.

3.5 Proposition 4: TEC(q) is convex in (0, ∞).

Let $X_1 = TEC_1$ (q) and $Y_1 = TEC_2$ (q), now it is required to show that, $F(X_1, Y_1) = uX_1 + (1-u) Y_1 \ \varepsilon\ (0, \infty)$ and $u\ \varepsilon\ [0,1]$ where $X_1\ \varepsilon\ (0, \infty)$ and $Y_1\ \varepsilon\ (0, \infty)$.

Proof:

First, we check at u =0

$F(X_1, Y1) = uX_1+ (1-u) Y1 = X_1\ \varepsilon\ (0, \infty)$

at u=1,

$F(X_1, Y1) = uX_1+ (1-u) Y1 = X_1\ \varepsilon\ (0, \infty)$

at $u \neq 0$; or $0 < u < 1$

$$\begin{aligned} F(X_1, Y1) &= u [A_m/q + B_m+C_m q+(1-u) Am^*/q + B_m^*+ C_m^*q \\ &= u[(A_m-A_m^*)/q + B_m-B_m^* + (C_m-C_m^*)/q + A_m^*/q + B_m^*+ C_m^*q \\ &= u [\{2[(T-Z)\ \Delta + cI_et^2\Delta^2/2\}/q+ (S + ^- Y + R - X)\Delta + A_m^*/q + B_m^*+C_m^*q \end{aligned}$$

Finally, it is clear that

$$0 < [(T-Z)\ \Delta + cI_et^2\Delta^2/2\}/q] < \infty, 0 < [S + ^- Y] < \infty$$

$0 < (R - X) < \infty$ and $0 < [A_m^*/q + B_m^* + C_m^* q] < \infty$

Hence, $uX_1 + (1-u) Y_1 \, \varepsilon \, (0, \infty)$.

Suppose the function TEC (q) defined by (2) is monotonically decreasing in an interval (V_{r-1}, V_r). It follows from the proof of proposition 2 and 3 that it must be decreasing in (V_{r-2}, V_{r-1}) and consequently in all the preceding intervals also. Further, if TEC (q) is monotonically increasing in an integral (V_{r-1}, V_r) then in all the following intervals also the function will be increasing. Thus, the existence and uniqueness of optimal order quantity is ensured. Moreover, the behaviour of TEC (q) at $(t\Delta)$ indicates whether the optimal order quantity is less than or equal to or greater than $(t\Delta)$

Remark:

It is quite interesting to note that if probabilistic demand and non-zero lead time turn to be deterministic and zero respectively, the present model reduces to model [13]. Moreover, if we consider single ware house (with infinite capacity) in model [13], the same reduces to model [3]. Thus, model [3] and [13] both can be considered as particular cases of present model.

3.6 An algorithm development

Step 1: Find n such that $V_{n-1} < t\Delta \leq V_n$

Step 2: If $A_n/C_n \neq (t\Delta)^2$, go to step 3, otherwise optimal order quantity $q^* = t\Delta$

Step 3: If $A_n/C_n > (t\Delta)^2$, go to step 4, otherwise find the smallest value of r, say k, such that $A_r/C_r \leq V_r^2$ and the optimal order quantity $q^* = \sqrt{(A_k/C_k)}$

Step 4: If $A_n^*/C_n^* \leq (t\Delta)^2 < V_n^2$ or $A_{n+1}^*/C_{n+1}^* \leq (t\Delta)^2 = V_n^2$ the optimal order quantity $q^* = t\Delta$. Otherwise find the smallest value of r, say k, for which $V_r^2 \geq A_r^*/C_r^*$ and then the optimal order quantity $q^* = \sqrt{(A_k^*/C_k^*)}$.

4. Numerical example

We shall now illustrate the above model by an example. Suppose the parameter values are: $\Delta = 5900$ / year,

C_0 = Rs.20.00, c= 18, $\sigma_0 = 0$, $\sigma_1 = 1$, b = ½, q' = 150, I_e = 10%, I_p = 18%

Warehouse capacities: $W_1 = 240$, $W_2 = 150$, $W_3 = 210$, $W_4 = 95$, $W_5 = \infty$.

H_1 = Rs.3.00, H_2 = Rs.3.25, H_3 = Rs.3.50, H_4 = Rs.3.75, H_5 = Rs.4.00.

The optimal order quantity for different values of t are given below for $H_3 = 3.50$.

Time (Months)

	0.95	0.96	0.97	0.975	0.976	0.977
q*	470	475	480	481.5	481.89	481.89

We observe that if $H_3 = 3.50$ then for $t \geq 0.976$ the optimal order quantity is 481.89 i.e. there is no use of going for higher order quantity than 481.89 for $t \geq 0.976$. At this stage, we examine whether this optimal order quantity in H_3 is sensitive to small change by considering the cases $H_3 = 3.45$ and $H_3 = 3.55$. We investigate the effect of changing H_3 slightly. The results are given in the following table.

Optimal Order Quantity (q*) for Various values of H_3 and t							
H_3 3.45	t	0.95	0.96	0.97	0.975	0.976	0.977
	q*	470	475	480	484.5	484.89	484.89
3.50	t	0.95	0.96	0.97	0.972	0.973	0.974
	q*	470	475	480	481.5	481.89	481.89
3.55	t	0.95	0.96	0.96	0.967	0.97	0.971
	q*	470	475	479	479.45	479.45	479.45

Here, it is observed that when H_3 = 3.45 and ($t \geq 0.976$), the optimal order quantity is 484.89 whereas when H_3 = 3.55 and ($t \geq 0.967$), it is 479.45. This indicates that optimal order quantity is quite sensitive to change in H_3. The table also reveals that as H_3 increases, the time (t) beyond which any credit will not alter the optimal order quantity (q*), keeps on decreasing. Hence availing further credit by increasing order quantity may not help.

5. Conclusion

The present study reveals that the stockist is encouraged to increase order size by extending a limited time credit facility and the developed probabilistic inventory model is more generalized model as it includes the models [3] and [13] as special cases. Numerical example indicates that the optimal order quantity is sensitive to the holding cost at 3rd warehouse.

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