Enhancing Accuracy in Population Mean Estimation with Advanced Memory Type Exponential Estimators

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Abstract

For a number of reasons, mean estimate is an essential sampling activity as it offers crucial information and forms the basis of statistical inference and judgement. In this study, we estimate the population mean using the Exponentially Weighted Moving Average (EWMA) statistic and provide generalized family of exponential estimators. The theoretical aspects of the suggested estimator are evaluated via rigorous mathematical derivations of the bias and mean square error (*MSE*), which are then compared to other exponential estimators that are already in use. Furthermore, a thorough simulation research is carried out to thoroughly assess the effectiveness and empirical performance of the suggested strategy. The results highlight how the estimator's effectiveness is significantly increased when both recent and historical data are used in tandem.

Keywords: Bias, Exponentially Weighted Moving Average (*EWMA*), Mean Square Error (*MSE*), Memory type estimator, Percent Relative Efficiency (*PRE*).

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1 Introduction

Utilizing supplementary information is a crucial tactic in survey sampling to increase estimators accuracy in calculating the population mean. Additional, easily accessible population data that is connected with the study variable and may be used to increase estimating accuracy and efficiency is referred to as auxiliary information.

The ratio estimator propounded by Cochran [5] is usually used when the study and auxiliary variable have a positive linear relationship. By taking advantage of the proportionality between the two variables, this estimator permits modifications that are consistent with their direct correlation. By taking into account the strength of positive correlation, the ratio estimator efficiently lowers variance and improves the estimate dependability .

On the other hand, the product estimator given by Robson [14] is better suitable when the linear connection is negative. This estimator makes adjustments that reflect the opposing trends of the study and auxiliary variables by taking advantage of their inverse connection. Despite the divergent directional trends, the product estimator guarantees more precise population mean predictions by taking into account the negative correlation.

The importance of auxiliary information in improving estimate methods in survey sampling is highlighted by the careful selection of these estimators based on the kind of correlation between the study and auxiliary variables.

Many authors [1, 4, 6, 7, 16, 17, 19, 21–25] have extensively utilized auxiliary information to refine and enhance the efficiency of estimators under various sampling designs. These contributions underscore the pivotal role of auxiliary variables in improving the accuracy and reliability of population parameter estimates, demonstrating their applicability across a wide array of methodological advancements and practical scenarios.

In recent decades, the systematic collection of data through time-scaled surveys has gained significant importance across various research fields, becoming essential for informed decision-making and effective policy formulation. Notable examples include the National Sample Survey (*NSS*) and the National Family Health Survey (*NFHS*), both conducted every five years by the Government of India. Additionally, the Annual Status of Education Report (*ASER*) and the Periodic Labour Force Survey (*PLFS*), conducted annually, provide critical insights into demographic, health, and educational trends over time. A significant challenge arises when conventional estimators are employed to estimate the population parameter from these time-scaled surveys. These estimators give ordinary results that fail to capture the complexity of the data, primarily due to their design for cross-sectional studies, which fails to account for the temporal trends inherent in longitudinal data. As a result, important changes over time, such as fluctuations in employment rates and trends in healthcare access, are overlooked, resulting in potentially misleading conclusions for policy-making.

To address these challenges, we utilize the EWMA statistic, which assigns exponentially decreasing weights to past observations. By placing greater emphasis on more recent data, EWMA facilitates a more dynamic analysis of trends. In this study, we explore the effectiveness of EWMA in estimating population parameters and propose a memory-type exponential estimator specifically designed for time-scaled surveys. Roberts [13] was the first to propose the idea of EWMA. Several authors [2, 3, 8–11, 15, 20] have utilized EWMA statistic to estimate population parameters in the context of time-scaled surveys. Their research emphasizes how important EWMA is for combining current and historical data, which improves estimating accuracy and efficiency in dynamic survey environments. Nonetheless, there is still a dearth of research on exponential estimators for time-scaled surveys. Numerous sampling methods and their uses have been extensively studied, but the particular use of exponential estimators in time-scaled surveys has not gotten as much attention. Since exponential estimators have the potential to increase the precision and effectiveness of population parameter estimation, particularly when taking into account the temporal dynamics of data collection in time-series or longitudinal surveys, this gap offers a chance for more research.

EWMA Statistic- The EWMA statistic is a memory-type statistic that enhances estimator efficiency by weighting past and present data. Roberts [13] was the first to introduce the EWMA statistic to observe the change in process mean and is given by

$$Z_i = \lambda \bar{y} + (1 - \lambda) Z_{i-1}$$

where $\bar{y}$ is the mean of current data, and $0 \leq \lambda \leq 1$ is the smoothing constant, which varies proportionally to the weight given to the latest data and is inversely proportional to the weight given to past value (information). Note that when λ takes the value 1, it means that all weight is given to the latest data, and in this case, the EWMA statistic is equal to $\bar{y}$. Here i denotes the number of samples, and Z_{i-1} denotes the past value (information). Here we assume the starting value of Z_{i-1} i.e., Z_0 is equal to zero.

The term "$exponentially\ weights$" means the weight λ decreases exponentially as the number of past data points increases. And

$$\mathrm{E}[Z_i] = \bar{Y} \text{ and } Var[Z_i] = \sigma_Y^2 \left(\frac{\lambda}{2-\lambda}\right)(1-(1-\lambda))^{2i}$$

where $\bar{Y}$ and σ_Y^2 is the mean and variance of the study variable respectively. And the limiting variance of Z_i is given by

$$Var[Z_i] = \sigma_Y^2 \left(\frac{\lambda}{2-\lambda}\right)$$

Now we briefly outline the rest of the manuscript. In Section 2, we reassess several existing estimators from the literature and derive the expression for their MSE. Section 3 introduces a class of memory-type exponential estimators for which we determine the minimum MSE. In Section 4, we conduct an extensive simulation study. Finally, Sections 5 present the conclusion of our study.

2 Review of Some Existing Estimators in Literature

First, we review several prominent estimators that have been extensively studied and applied in the literature and then modify them into memory-type estimators to improve their efficiency.

Let Y and X be the study and auxiliary variables, respectively, within a population $U = \{U_1, U_2, \ldots, U_N\}$ having N units. Let $\bar{y}$ and $\bar{x}$ denote the sample means of the study variable and the auxiliary variable, respectively. Additionally, let

$$Z_i = \lambda\bar{y} + (1-\lambda)Z_{i-1} \quad (1)$$

$$Q_i = \lambda\bar{x} + (1-\lambda)Q_{i-1} \quad (2)$$

be the EWMA statistic for study and auxiliary variables, respectively. Based on the above population, a summary of several related existing estimators along with their *MSE* is provided below:

(a) The classical ratio estimator suggested by Cochran [5] is

$$\hat{\bar{y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X}$$

Further Noor-ul Amin [8] suggested the memory type ratio estimator as follows:

$$\hat{\hat{y}}_{mri} = \frac{\bar{Z}_i}{\bar{Q}_i}\bar{X} \tag{3}$$

The approximate *MSE* of $\hat{\hat{y}}_{mri}$ is given by

$$MSE(\hat{\hat{y}}_{mri}) = \left(\frac{\lambda}{2-\lambda}\right) f_1 \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_x C_y) \tag{4}$$

where $f_1 = \frac{1}{n} - \frac{1}{N}$, C_y and C_x represent the coefficients of variation for the study and auxiliary variables, respectively, and ρ is the correlation coefficient between the study and auxiliary variables.

(b) Regression estimator suggested by Watson [22] is as follows:

$$\hat{\hat{y}}_{reg} = \bar{y} + b(\bar{X} - \bar{x})$$

where b is the regression coefficient. By utilizing (1) and (2), in $\hat{\hat{y}}_{reg}$ the memory-type regression estimator is given as:

$$\hat{\hat{y}}_{mrgi} = Z_i + b(\bar{X} - Q_i) \tag{5}$$

The approximated *MSE* of $\hat{\hat{y}}_{mrgi}$ is given below

$$MSE(\hat{\hat{y}}_{mrgi}) = \left(\frac{\lambda}{2-\lambda}\right) f_1 \bar{Y}^2 C_y^2 \left(1 - \rho^2\right) \tag{6}$$

(c) The exponential ratio type estimator suggested by Bahl and Tuteja [4] is given by

$$\hat{\hat{y}}_{ex} = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)$$

Now, employing (1) and (2) in the above expression, the memory type exponential ratio estimator is given as:

$$\hat{\hat{y}}_{mexi} = Z_i \exp\left(\frac{\bar{X} - Q_i}{\bar{X} + Q_i}\right) \tag{7}$$

and we obtain the approximate *MSE* of $\hat{\hat{y}}_{mexi}$, which is as follows:

$$MSE(\hat{\hat{y}}_{mexi}) = \left(\frac{\lambda}{2-\lambda}\right) f_1 \bar{Y}^2 \left[C_y^2 + C_x^2 \left(\frac{1}{4} - \frac{\rho C_y}{C_x}\right)\right] \tag{8}$$

3 Proposed Class of Memory Type Exponential Estimators

Now, in this section, we propose a class of memory-type exponential estimators that introduce a novel approach to improving estimation accuracy. These estimators are designed to efficiently incorporate past information, potentially resulting in lower *MSE* and higher *PRE* compared to existing methods.

Suppose Y and X are the study and auxiliary variables, respectively. The exponential type estimator given by [18] is

$$t_{pe} = \left\{\vartheta_1 \bar{y} + \vartheta_2 \left(\frac{\bar{y}}{\bar{x}}\right) \bar{X}\right\} \exp\left[\frac{\alpha(\bar{X} - \bar{x})}{\alpha(\bar{X} + \bar{x}) + 2\beta}\right]$$

where $\bar{y}$ and $\bar{x}$ are the sample mean of study and auxiliary variable, $\bar{X}$ is the population mean of auxiliary variable, ϑ_1 and ϑ_2 denote approximately chosen constants intended to minimize $MSE(t_{pe})$, while α and β are real constants.

Now, employing (1) and (2), in t_{pe} the memory type exponential estimator is given as follows:

$$t_{mpei} = \left\{\vartheta_1 Z_i + \vartheta_2 \left(\frac{Z_i}{Q_i}\right) \bar{X}\right\} \exp\left[\frac{\alpha(\bar{X} - Q_i)}{\alpha(\bar{X} + Q_i) + 2\beta}\right] \tag{9}$$

where ϑ_1 and ϑ_2 represent approximately chosen constants aimed at minimizing $MSE(t_{mpei})$, α, β are real constants, and $\bar{X}$ is the population mean (which is known in advance) of auxiliary variable.

We use the Taylor series expansion to calculate the minimum *MSE* of the estimator t_{mpei} up to the second-order approximation, using the terms listed

Table 1 Members of the proposed class of estimator for different value of ϑ_1 and ϑ_2.

α	β	ϑ_1	ϑ_2	Estimators
1	1	ϑ_1	ϑ_2	$t_{mpei_1} = \left\{\vartheta_1 Z_i + \vartheta_2 \left(\frac{Z_i}{Q_i}\right) \bar{X}\right\} \exp\left[\frac{\bar{X} - Q_i}{\bar{X} + Q_i + 2}\right]$
1	-1	ϑ_1	ϑ_2	$t_{mpei_2} = \left\{\vartheta_1 Z_i + \vartheta_2 \left(\frac{Z_i}{Q_i}\right) \bar{X}\right\} \exp\left[\frac{\bar{X} - Q_i}{\bar{X} + Q_i - 2}\right]$
1	0	ϑ_1	ϑ_2	$t_{mpei_3} = \left\{\vartheta_1 Z_i + \vartheta_2 \left(\frac{Z_i}{Q_i}\right) \bar{X}\right\} \exp\left[\frac{\bar{X} - Q_i}{\bar{X} + Q_i}\right]$
0	1	ϑ_1	ϑ_2	$t_{mpei_4} = \left\{\vartheta_1 Z_i + \vartheta_2 \left(\frac{Z_i}{Q_i}\right) \bar{X}\right\}$

below:

$$\zeta_0 = \frac{Z_i - \bar{Y}}{\bar{Y}}, \quad \zeta_1 = \frac{Q_i - \bar{X}}{\bar{X}} \tag{10}$$

such that

$$\mathrm{E}[\zeta_0] = \mathrm{E}[\zeta_1] = 0 \tag{11}$$

$$\mathrm{E}[\zeta_0^2] = f_1 \frac{\mathrm{Var}(Z_i)}{\bar{Y}^2} = \left(\frac{\lambda}{2-\lambda}\right) f_1 C_y^2, \tag{12}$$

$$\mathrm{E}[\zeta_1^2] = f_1 \frac{\mathrm{Var}(Q_i)}{\bar{X}^2} = \left(\frac{\lambda}{2-\lambda}\right) f_1 C_x^2, \tag{13}$$

$$\mathrm{E}[\zeta_1\zeta_1] = f_1 \frac{Cov(Z_i, Q_i)}{\bar{Y}\bar{X}} = \left(\frac{\lambda}{2-\lambda}\right) f_1 \rho C_y C_x \tag{14}$$

Utilizing equation (10) in (9), we have

$$t_{mpei} = \left\{\vartheta_1 \bar{Y}(1+\zeta_0) + \vartheta_2 \frac{\bar{Y}(1+\zeta_0)}{(1+\zeta_1)}\right\} \exp\left[\frac{-\alpha\bar{X}\zeta_1}{\alpha\bar{X}\zeta_1 + 2(\beta+\alpha\bar{X})}\right]$$

we can also write the above equation as:

$$t_{mpei} = \left\{\vartheta_1 \bar{Y}(1+\zeta_0) + \vartheta_2 \frac{\bar{Y}(1+\zeta_0)}{(1+\zeta_1)}\right\} \exp\left[\frac{-\gamma\zeta_1}{1+\gamma\zeta_1}\right]$$

where $\gamma = \frac{\alpha\bar{X}}{2(\beta+\alpha\bar{X})}$. Now, by subtracting $\bar{Y}$ from both sides of the above equation, we obtain

$$\begin{aligned} t_{mpei} - \bar{Y} = \bar{Y} \bigg[& \vartheta_1 \left\{1 - \gamma\zeta_0^2 + \frac{3}{2}\gamma^2\zeta_1^2\right\} \\ & + \vartheta_2 \left\{1 + (1 + \gamma + \frac{3}{2}\gamma^2)\zeta_1^2 - (1+\gamma)\zeta_0\zeta_1\right\} - 1\bigg] \end{aligned} \tag{15}$$

Employing expectation on both sides of the Equation (15), and using (12), (13), (14), we have

$$\begin{aligned} Bias(t_{mpei}) = {} & \bar{Y}\vartheta_1 \left\{1 - f_1\left(\frac{\lambda}{2-\lambda}\right)\left(\gamma C_y^2 - \frac{3}{2}\gamma^2 C_x^2\right)\right\} \\ & + \bar{Y}\vartheta_2 \bigg[\bigg\{1 + f_1\left(\frac{\lambda}{2-\lambda}\right) \\ & \left((1+\gamma+\frac{3}{2}\gamma^2)C_x^2 - (1+\gamma)\rho C_x C_y\right)\bigg\} - 1\bigg] \end{aligned} \tag{16}$$

By squaring and taking expectation on both sides of (15) and applying (11), (12), (13), and (14), we get:

$$MSE(t_{mpei}) = A_m\vartheta_1^2 + B_m\vartheta_2^2 + 2C_m\vartheta_1\vartheta_2 + 2D_m\vartheta_1 + 2E_m\vartheta_2 + F_m \quad (17)$$

where

$$A_m = \bar{Y}^2\left[1 + f_1\left(\frac{\lambda}{2-\lambda}\right)\{C_y^2 + C_x^2 - 2\rho C_y C_x\}\right]$$

$$B_m = \bar{Y}^2\left[1 + f_1\left(\frac{\lambda}{2-\lambda}\right)\{C_y^2 + (3 + 4\gamma + 4\gamma^2)C_x^2 \right.$$
$$\left. -4(1+\gamma)\rho C_y C_x\}\right]$$

$$C_m = \bar{Y}^2\left[1 + f_1\left(\frac{\lambda}{2-\lambda}\right)\{C_y^2 + (1 + 2\gamma + 4\gamma^2)C_x^2 \right.$$
$$\left. -2(1+2\gamma)\rho C_y C_x\}\right]$$

$$D_m = -\bar{Y}^2\left[1 + f_1\left(\frac{\lambda}{2-\lambda}\right)\left\{\frac{3}{2}\gamma^2 C_x^2 - \rho C_y C_x\right\}\right]$$

$$E_m = -\bar{Y}^2\left[1 + f_1\left(\frac{\lambda}{2-\lambda}\right)\left\{\left(1 + \gamma + \frac{3}{2}\gamma^2\right)C_x^2 - (1-\gamma)\rho C_y C_x\right\}\right]$$

$$F_m = \bar{Y}^2$$

To minimize the *MSE* of the estimator t_{mpei}, we differentiate equation (17) with respect to ϑ_1 and ϑ_2, we have

$$\vartheta_1 = \frac{B_m D_m - C_m E_m}{C_m^2 - A_m B_m} = \vartheta_1^\star \quad (18)$$

$$\vartheta_2 = \frac{A_m E_m - C_m D_m}{C_m^2 - A_m B_m} = \vartheta_2^\star \quad (19)$$

Now, utilizing $\vartheta_1^\star$ and $\vartheta_2^\star$ in equation (17), we obtain the expression for minimum *MSE* of t_{mpei}

$$MSE_{min}(t_{mpei}) = A_m\vartheta_1^{\star 2} + B_m\vartheta_2^{\star 2} + 2C_m\vartheta_1^\star\vartheta_2^\star + 2D_m\vartheta_1^\star$$
$$+ 2E_m\vartheta_2^\star + F_m. \quad (20)$$

4 Simulation Studies

A comprehensive simulation study was conducted to evaluate the effectiveness of the proposed memory-type estimators. The Mean Squared Error (*MSE*) and Percent Relative Efficiency (PRE) of both the proposed and existing estimators, relative to the usual estimator $\bar{y}$, were calculated using the following formulas, based on 10,000 replications:

$$MSE(t_j) = \frac{1}{10000}\sum_{j=1}^{10000}(t_j - \bar{Y})^2 \tag{21}$$

and

$$PRE(t_j, \bar{y}) = \frac{MSE(\bar{y})}{MSE(t_j)} \tag{22}$$

where $t_j = t_{mri}, t_{mrgi}, t_{mexi}, t_{mpei_1}, t_{mpei_2}, t_{mpei_3}, t_{mpei_4}$ for $j = 1, 2, 3, 4, 5, 6, 7$ respectively.

The PRE of the estimators is calculated at various levels of correlation $\rho = (0.75, 0.80, 0.85, 0.90, 0.95)$ and weight parameter $\lambda = (0.10, 0.25, 0.50, 0.75, 0.95)$ using the algorithm given by [12]:

(i) Generate two independent population of size $N = 5000$ such that $X = N(10, 4)$ and $Z = N(10, 4)$.
(ii) Set $Y = \rho X + \sqrt{1-\rho^2}Z$ where ρ is the correlation between X and Y, and take the value for λ.
(iii) Select 10000 samples of sizes $n = 50, 100, 200, 300, 500$ respectively. And compute the estimator for each 10000 samples.
(iv) Compute the *MSE* for each sample size for each estimator using (21).
(v) Obtained the relative efficiencies for each sample using (22).

5 Discussion and Results

Tables 2 and 3 represents the PRE of the existing and the proposed estimators relative to usual estimator $\bar{y}$, with smoothing constant $\lambda = 0.10, 0.25, 0.50$ and 0.95, across different values of ρ and n. Key findings from Tables 2 and 3 are:

(i) As λ (smoothing constant) decreases from 0.95 to 0.10 for any fixed value of ρ $(0.75 \leq \rho \leq 0.95)$ the PRE of the proposed estimators $t_{mpei_j}, j = 1, 2, 3, 4$ increases. Here λ indicate the weight assign to current information so if we take $\lambda = 1$ i.e. we use only current

Table 2 PRE of estimators $t_{mri}, t_{mrgi}, t_{mexi}, t_{mpei_1}, t_{mpei_2}, t_{mpei_3}, t_{mpei_4}$ relative to usual estimator $\bar{y}$, with smoothing constant $\lambda = 0.10, 0.25$, across different values of ρ.

		$\lambda = 0.10$							$\lambda = 0.25$						
ρ	n	t_{mri}	t_{mrgi}	t_{mexi}	t_{mpei_1}	t_{mpei_2}	t_{mpei_3}	t_{mpei_4}	t_{mri}	t_{mrgi}	t_{mexi}	t_{mpei_1}	t_{mpei_2}	t_{mpei_3}	t_{mpei_4}
	50	130.339	259.769	241.379	259.803	259.815	259.808	259.777	124.519	248.9633	236.981	248.982	248.9865	248.984	248.9752
	100	130.236	256.483	240.919	256.499	256.504	256.501	256.486	124.446	245.8732	236.565	245.8819	245.884	245.8829	245.8789
0.75	200	130.2458	254.9266	240.6422	254.9345	254.9371	254.9356	254.9284	124.4498	244.3562	236.2858	244.3605	244.3615	244.3609	244.3589
	300	130.4292	254.8418	240.9131	254.8471	254.8488	254.8478	254.843	124.6254	244.2817	236.5581	244.2845	244.2852	244.2848	244.2835
	500	130.4138	254.2836	240.7363	254.2866	254.2875	254.287	254.2843	124.615	243.7464	236.3703	243.7481	243.7484	243.7482	243.7475
	50	158.2099	315.3172	282.7581	315.3644	315.3808	315.3716	315.3266	151.2466	302.4025	278.9843	302.4275	302.4339	302.4303	302.4168
	100	158.0578	311.273	282.2054	311.2949	311.3026	311.2983	311.2773	151.1315	298.5969	278.475	298.6085	298.6115	298.6099	298.6037
0.80	200	158.0433	309.3339	281.8465	309.3446	309.3483	309.3462	309.33	151.1121	296.7074	278.1128	296.7132	296.7146	296.7138	296.7108
	300	158.2765	309.2517	282.1398	296.7108	309.2613	309.2599	309.2532	151.3367	296.6392	278.4107	296.643	296.644	296.6434	296.6414
	500	158.236	308.5317	281.9107	308.5358	308.5372	308.5364	308.5325	151.3028	295.9477	278.1643	295.9498	295.9504	295.9501	295.9489
	50	204.6092	407.7925	338.1489	407.864	407.8901	407.8755	407.8044	195.8653	391.6131	335.8177	391.6501	391.6604	391.6547	391.6315
	100	204.3758	402.49	337.507	402.5232	402.5355	402.5286	402.4954	195.6814	386.6162	335.2139	386.6335	386.6383	386.6356	386.6249
0.85	200	204.3206	399.9111	337.0577	399.9273	399.9333	399.93	399.9138	195.6229	384.104	334.7582	384.1125	384.1148	384.1135	384.1083
	300	204.6336	399.8276	337.3538	399.8384	399.8423	399.8401	399.8294	195.9262	384.0404	335.0621	384.046	384.0476	384.0467	384.0432
	500	204.5502	398.8362	337.07	398.8423	398.8446	398.8433	398.8372	195.8521	383.0859	334.7517	383.0891	383.09	383.0895	383.0875
	50	297.0771	592.0839	410.9774	592.2381	592.2666	592.2381	592.1006	285.0404	569.9099	411.4358	569.9761	569.9964	569.9851	569.9365
	100	296.6856	584.2814	410.3027	584.3428	584.3668	584.3534	584.2891	284.7227	562.5388	410.7865	562.5697	562.5792	562.5739	562.5514
0.90	200	296.5472	580.4236	409.7885	580.4537	580.4653	580.4588	580.4274	284.5846	558.7797	410.2613	558.7948	558.7994	558.7968	558.7858
	300	297.0134	580.3257	410.0346	580.3455	580.3531	580.3489	580.3282	285.0392	558.7133	410.5154	558.7233	558.7263	558.7246	558.7173
	500	296.8422	578.789	409.7203	578.8003	578.8047	578.8022	578.7904	284.8825	557.2291	410.1636	557.2348	557.2365	557.2355	557.2314
	50	572.666	1141.341	491.1003	1141.74	1141.906	1141.813	1141.372	551.6907	1103.051	495.368	1103.242	1103.309	1103.271	1103.102
	100	571.8166	1126.114	490.5995	1126.301	1126.379	1126.335	1126.128	550.984	1088.603	494.8742	1088.692	1088.724	1088.706	1088.627
0.95	200	571.4236	1118.432	490.1682	1118.523	1118.56	1118.539	1118.439	550.6043	1081.107	494.4319	1081.151	1081.166	1081.157	1081.119
	300	572.3255	1118.25	490.2576	1118.31	1118.335	1118.321	1118.255	551.4904	1080.992	494.5201	1081.02	1081.03	1081.025	1080.999
	500	571.8849	1115.073	490.0197	1115.107	1115.121	1115.113	1115.076	551.0793	1077.909	494.2434	1077.925	1077.931	1077.928	1077.913

Table 3 PRE of estimators $t_{mri}, t_{mrgi}, t_{mexi}, t_{mpei_1}, t_{mpei_2}, t_{mpei_3}, t_{mpei_4}$ relative to usual estimator $\bar{y}$, with smoothing constant $\lambda = 0.50, 0.95$, across different values of ρ.

		$\lambda = 0.50$							$\lambda = 0.95$						
ρ	n	t_{mri}	t_{mrgi}	t_{mexi}	t_{mpei_1}	t_{mpei_2}	t_{mpei_3}	t_{mpei_4}	t_{mri}	t_{mrgi}	t_{mexi}	t_{mpei_1}	t_{mpei_2}	t_{mpei_3}	t_{mpei_4}
	50	119.902	240.574	233.365	240.613	240.614	240.613	240.639	119.377	239.907	233.025	240.01	240.012	240.011	240.08
	100	119.903	237.523	232.942	237.542	237.542	237.542	237.554	119.481	236.841	232.621	236.901	236.901	236.901	236.934
0.75	200	119.872	235.995	232.658	236.004	236.005	236.005	236.01	119.465	235.322	232.334	235.347	235.347	235.347	235.363
	300	120.048	235.936	232.946	235.942	235.942	235.942	235.946	119.6493	235.274	232.631	235.281	235.29	235.281	235.301
	500	120.051	235.436	232.764	235.439	235.439	235.439	235.441	119.659	234.767	232.448	234.776	234.776	234.776	234.782
	50	145.726	292.387	275.871	292.437	292.439	292.438	292.466	145.097	291.596	275.595	291.721	291.733	291.731	291.808
	100	145.694	288.613	275.327	288.637	288.638	288.637	288.651	145.188	287.808	275.049	287.873	287.874	287.874	87.91
0.80	200	145.635	286.716	274.971	286.727	286.728	286.727	286.734	145.147	285.911	274.687	285.942	285.943	285.943	285.961
	300	145.861	286.669	275.281	286.677	286.677	286.677	286.681	145.385	285.879	275.017	285.899	285.891	285.891	285.911
	500	145.845	286.021	275.046	286.026	286.026	286.026	286.028	145.376	285.223	274.771	285.235	285.235	285.2353	285.242
	50	188.936	379.086	333.901	379.156	379.151	379.158	379.188	188.146	378.108	333.766	378.296	378.304	378.299	378.383
	100	188.841	374.103	333.223	374.137	374.139	374.138	374.153	188.211	373.094	333.049	373.185	373.189	373.187	373.227
0.85	200	188.746	371.589	332.789	371.605	371.606	371.606	371.613	188.131	370.581	332.606	370.625	370.627	370.626	370.645
	300	189.053	371.557	333.111	371.568	371.568	371.568	371.572	188.455	370.561	332.949	370.599	370.591	370.599	370.611
	500	189.003	370.661	332.804	370.668	370.668	370.668	370.67	188.414	369.664	332.621	369.68	369.681	369.68	369.688
	50	275.507	552.782	411.883	552.898	552.906	552.902	552.9313	274.41	551.471	412.015	551.78	551.801	551.789	551.874
	100	275.312	545.381	411.101	545.437	545.441	545.439	545.453	274.425	543.998	411.147	544.147	544.157	544.151	544.191
0.90	200	275.118	541.633	410.629	541.651	541.662	541.661	541.667	274.267	540.253	410.654	540.325	540.33	540.327	540.347
	300	275.583	541.618	410.911	541.636	541.637	541.636	541.641	274.757	540.272	410.947	540.319	540.323	540.321	540.333
	500	275.464	540.221	410.543	540.231	540.232	540.232	540.234	274.651	538.859	410.571	538.886	538.888	538.887	538.894
	50	535.071	1073.594	499.076	1073.885	1073.919	1073.9	1073.887	533.135	1071.42	499.56	1072.192	1072.278	1072.23	1072.209
	100	534.566	1058.953	498.437	1059.092	1059.108	1059.099	1059.092	532.995	1056.565	498.775	1056.935	1056.976	1056.953	1056.942
0.95	200	534.108	1051.514	498.0625	1051.581	1051.588	1051.584	1051.581	532.611	1049.14	498.385	1049.319	1049.338	1049.327	1049.321
	300	535.023	1051.509	498.164	1051.552	1051.557	1051.555	1051.552	533.578	1049.208	498.488	1049.324	1049.337	1049.329	1049.325
	500	534.687	1048.594	497.859	1048.619	1048.622	1048.62	1048.619	533.269	1046.259	498.176	1046.325	1046.333	1046.328	1046.326

information then our proposed memory type exponential estimator is equal to the estimator t_{pe}.

(ii) Increasing the correlation coefficient between the study and the auxiliary variable results in a rise in the PRE of the estimators. This is true regardless of the values of λ and n. It may be inferred from this that the effectiveness of the estimators improves as the strength of the association between the study and the auxiliary variable increases (that is, as ρ increases). When the value of ρ is larger, it shows that the auxiliary variable offers more relevant information for predicting the study variable, which ultimately results in more accurate predictions. This information is used more effectively by the suggested estimators, which ultimately leads to an increase in PRE values.

(iii) As the sample size n rises, notably for the values $n = 50, 100, 200, 300,$ and 500, the proposed class of estimators has a PRE that is higher than that of the current estimators. This is the case even when the values of λ and ρ remain the same. The implication of this is that the proposed estimators demonstrate superior efficiency in utilizing the information provided by both the study and auxiliary variables as more data points become available. Furthermore, with a larger sample size, the estimators are able to better capture the underlying relationships and reduce the variability in the estimates. This better performance leads to a higher PRE, which indicates that the suggested estimators are more effective in terms of accuracy when compared to the alternatives that are currently available.

According to Figure 1, which depicts the influence of smoothing constant λ on the PRE of the suggested estimators, the sample size is set at $n = 200$, and the coefficient of correlation is $\rho = 0.90$. When the value of λ grows from zero to one, we find that the PRE of the suggested class of estimators rapidly drops. This is something that we see. The fact that PRE gradually decreases as the value of λ grows suggests that the suggested estimators perform better as they include more information from the past. This is because they become less susceptible to noise and fluctuations in the data that is currently being used.

According to Figure 2, which depicts the influence of correlation coefficient ρ on the PRE of the suggested estimators, the sample size is set at $n = 200$, and the smoothing constant is $\lambda = 0.50$. When the correlation coefficient ρ is increased, the suggested class of estimators experiences a rise in the PRE. The implication of this is that the suggested estimators become more effective as the linear connection between the study and auxiliary

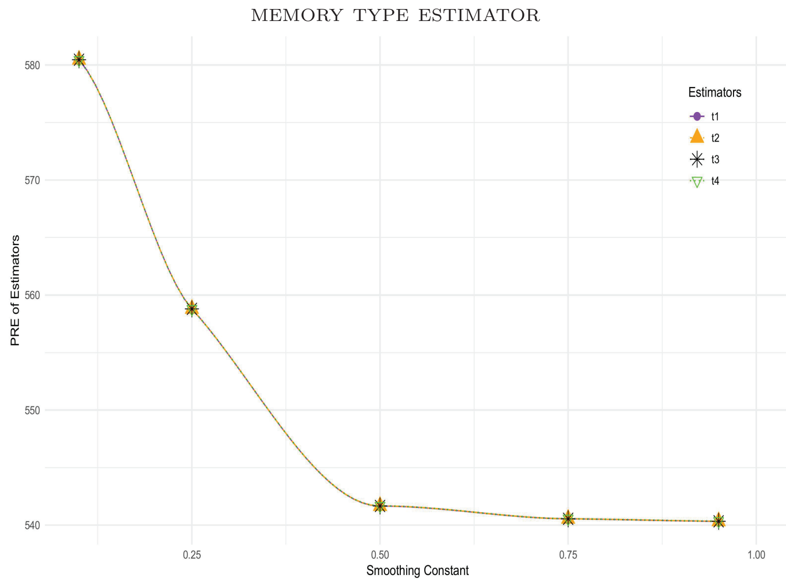

Figure 1 Effect of smoothing constant λ on PRE of estimators ti (here ti, i=1,2,3,4 used for notation of the estimators $t_{mpei_1}, t_{mpei_2}, t_{mpei_3}, t_{mpei_4}$ respectively.)

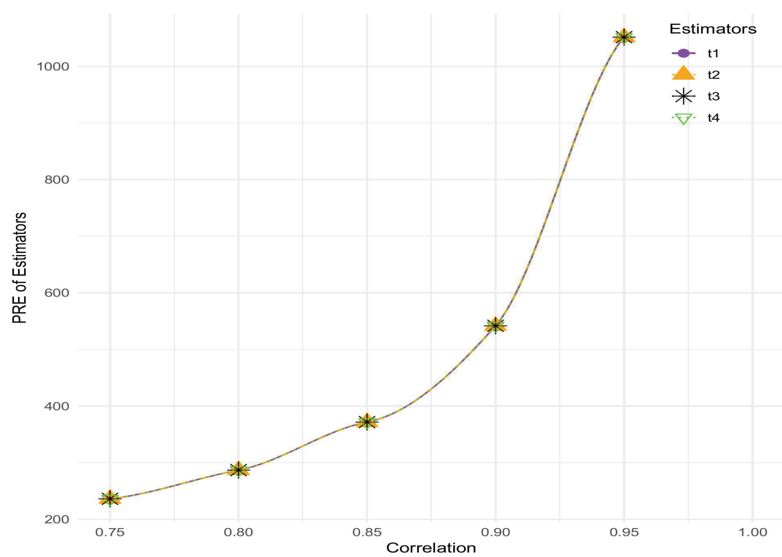

Figure 2 Effect of correlation ρ on PRE of estimators ti (here ti, i=1,2,3,4 used for notation of the estimators $t_{mpei_1}, t_{mpei_2}, t_{mpei_3}, t_{mpei_4}$ respectively.)

variables becomes stronger (that is, as ρ increases). When ρ is larger, it suggests that the auxiliary variable gives more pertinent information for predicting the study variable. This enables the estimator to make better use of the data that is accessible to them. Because of this, the performance of the estimator is enhanced, which ultimately results in a greater PRE percentage.This pattern demonstrates that a larger correlation between the study and auxiliary variables helps the estimator to attain better accuracy.

6 Conclusion

In this study, we aimed to enhance the efficiency of estimators by leveraging the concept of EWMA statistic. For the purpose of accomplishing this objective, we developed a family of estimators that include the EWMA statistic. Furthermore, in order to assess the effectiveness of these estimators, we carried out a comprehensive simulation research. The results of this investigation are shown in Tables 2 and 3 to illustrate the findings. It is obvious, after doing an analysis of the data included in these tables, that the suggested category of estimators consistently displays greater efficiency when compared to other established estimators, such as $\hat{\hat{y}}_{mei}$, $\hat{\hat{y}}_{mrgi}$, and $\hat{\hat{y}}_{mexi}$. Based on these results, we strongly suggest that our suggested family of estimators be used for the purpose of estimating population parameters since they provide a higher level of efficiency in comparison to the approaches that are already in use. Additionally, the scope of our research might be broadened by investigating other sampling methods, such as cluster or stratified sampling, and by using our estimators to estimate a wider variety of population characteristics, such as variances, proportions, or regression coefficients. This would allow us to investigate a wider range of population parameters. Furthermore evaluating the adaptability and robustness of the suggested estimators in a variety of statistical settings will be made easier with the assistance of this expansion.

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Biographies

Poonam Singh is a dedicated academician and researcher in the field of Statistics. She earned her Ph.D. in 2020 from Banaras Hindu University. With over eight years of experience in teaching and research, Dr. Singh specializes in modeling and estimating unknown population parameters in survey sampling, with a focus on addressing non-response and measurement errors. Currently serving in the Department of Statistics, Banaras Hindu University, she has published around 15 research articles in indexed journals, showcasing her contributions to the field. Dr. Singh is passionate about fostering collaboration and has been actively involved in international research initiatives. A skilled educator, Dr. Singh has a strong commitment to undergraduate and postgraduate teaching, inspiring future statisticians through her expertise and enthusiasm for the subject.

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