A Piecewise Smooth Approach to Modeling Innovation Adoption Under Time-Varying External Influences

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Abstract

Innovation diffusion modeling plays a crucial role in understanding how new technologies, products, or ideas spread through a population over time. Classical approaches such as the Bass model assume smooth and continuous adoption patterns, which often fail to capture abrupt changes caused by market dynamics, technological disruptions, or policy interventions. This study develops a piecewise smooth diffusion framework that extends the Bass innovation diffusion model to incorporate random shifts across different time intervals. The framework introduces modulation functions that allow both gradual transitions and abrupt perturbations in adoption rates, thereby reflecting the non-linear dynamics of real-world diffusion. Stability

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analysis is conducted to examine the robustness of the system. The model is applied to historical datasets on cassette sales, compact discs, and physical video records. Empirical evaluation demonstrates that the piecewise approach provides superior fitting accuracy compared with standard Bass formulations, while also reducing parameter estimation errors. The findings highlight the value of modeling random shifts in diffusion processes, offering new insights for understanding technology substitution and for designing adaptive marketing and policy strategies.

Keywords: Adoption rates, bass model, innovation diffusion process, piecewise smooth function.

1 Introduction

Innovation diffusion, the process by which new products and technologies spread within a population, has long been studied as a cornerstone of marketing science and innovation research. Classical theories emphasize a gradual, cumulative adoption pattern, where innovators and early adopters initiate a trajectory that ultimately reaches the majority of the market (Rogers, 2010). The Bass model (Bass, 1969) formalized this idea mathematically, generating the well-known S-shaped adoption curve that has since been widely applied across industries ranging from consumer goods to high technology.

Yet, real-world adoption rarely unfolds as a smooth, monotonic process. The phenomenon of “rise, fall … and rise again” is observed in numerous industries, reflecting growth phases punctuated by periods of decline, stagnation, or resurgence. Sales trajectories of durable goods, consumer electronics, and cultural products often reveal multiple waves of adoption, corresponding to successive cohorts of consumers, technological substitutions, or external disruptions. For example, Golder and Tellis (2004) demonstrated that only a minority of product categories conform strictly to the classical S-curve, with many instead following cycle–recycle or scalloped shapes. Similarly, Chandrasekaran and Tellis (2007) documented complex, multi-peak patterns in the life cycles of consumer technologies, underscoring the limitations of models that assume fixed, smooth adoption dynamics.

These deviations arise from a wide array of influences. Internal drivers such as psychological readiness, organizational resources, perceived usefulness, and innovativeness remain crucial generally for determining the underlying pace of adoption (Agarwal et al., 2018; Rahman & Thill, 2023). However, external factors including policy interventions, economic shocks, regulatory reforms, social disruptions, mass media exposure and marketing strategies accelerate, decelerate, or reverse adoption trajectories. Empirical models and case studies confirm that external shocks or interventions frequently explain the "bumps" or "waves" in real-world diffusion patterns (Mahajan & Muller, 1990; Peres, Muller, & Mahajan, 2010). For instance, Coccia (2020) highlights how institutional reforms and democratization shape innovation diffusion across nations. In the context of green technologies, regulatory pressure and subsidies significantly affect adoption rates, as shown in environmental policy studies (e.g., Li & Rao, 2023; Anand et al., 2025). More recently, electric vehicle adoption has surged in markets where governments introduced incentives and infrastructure investment (Pti, 2024), while industries such as gaming experienced abrupt declines in physical product sales during the COVID-19 pandemic due to supply chain disruptions (Sony, 2024). These cases illustrate that adoption dynamics are deeply embedded in – and frequently perturbed by – external environments.

Traditional diffusion extensions have attempted to capture such effects by explicitly linking adoption to decision variables like advertising, pricing, and promotion. These adjustments are effective levers for shaping product adoption but their effectiveness depends on consumer attitudes, market context, and the quality of information provided (Darke & Chung, 2005; Zhao et al., 2021; Niu et al., 2024). Further, these approaches only address "explainable variations" under managerial control – data dependent and remain limited in capturing abrupt and random adoption shifts. In contrast, unanticipated and exogenous changes – ranging from macroeconomic crises to geopolitical events – remain difficult to incorporate into conventional models. Addressing this gap requires a framework that can dynamically adjust adoption intensity as external conditions evolve.

The present study contributes to this need by proposing an extended diffusion model that incorporates a modulation function to account for external influences on adoption. By allowing adoption parameters to vary over time – whether gradually or abruptly – the model captures turning points, multi-peak cycles, and non-monotonic behaviours in product life cycles (PLCs). Both constant and continuous piecewise functional forms are implemented to distribute adoption momentum smoothly across phases. This extension not only enhances descriptive accuracy but also provides managers and policymakers with a more realistic tool to anticipate shifts in market dynamics. The specific objectives of this paper are as follows:

- To develop a piecewise smooth innovation diffusion model that systematically incorporates random shifts due to factors.

- To empirically validate the model using historical datasets on cassette sales, compact discs, and physical video records.
- To compare model performance with classical Bass and related extensions, evaluating improvements in accuracy and parameter stability.
- To derive managerial and policy implications for forecasting characterized by uncertainty.

The remainder of the paper is organized as follows: Section 2 reviews the literature on innovation diffusion and piecewise modeling approaches. Section 3 presents the modeling framework and stability analysis. Section 4 provides empirical illustrations. Section 5 discusses results, implications, and future extensions whereas conclusion is framed in Section 6.

2 Literature Review

The study of innovation diffusion originates from the seminal work of Rogers (2010), who conceptualized how innovations spread among populations. Building on this foundation, Bass (1969) introduced the Bass Innovation Diffusion Model (IDM), which explains adoption as a function of innovation (external influence) and imitation (internal influence). While the Bass model has become one of the most widely applied tools for forecasting adoption trajectories, several limitations restrict its applicability.

First, the assumption of population homogeneity oversimplifies real-world diffusion, where adoption often unfolds through clustered or network-driven interactions rather than uniform mixing (Bemmaor & Lee, 2002; Fibich & Golan, 2023; Jha et al., 2008). Second, the assumption of static market potential fails to account for demographic shifts, technological advances, or policy interventions that alter the size of the addressable market over time (Wang et al., 2017; Zhang et al., 2020). Third, the model is less suited for services or repeat-purchase contexts involving cycles of adoption, dis-adoption, and re-adoption (Saeed et al., 2020; Øverby et al., 2022). Fourth, it excludes the influence of marketing and external shocks – such as advertising, pricing, online reviews, or macroeconomic conditions – that are increasingly central to adoption dynamics (Zhang et al., 2020; Cosguner et al., 2022). Finally, parameter estimation remains a persistent challenge, as reliable estimates of market potential and adoption coefficients are often unstable when data are limited, truncated, or perturbed by shocks (Kim et al., 2015; Massiani, 2015; Liang, 2021). Among these limitations, parameter instability and the exclusion of external drivers are particularly critical, underscoring the need for extended models that better capture the complexity and dynamism of modern innovation environments.

Several extensions and adaptations of the Bass innovation diffusion model have been developed to incorporate external drivers such as advertising, price, government incentives, and distribution, enhancing the model's ability to capture real-world adoption dynamics. Nonlinear and Generalized Bass models introduce these external influences as time-varying parameters, allowing for a more realistic representation of marketing campaigns, pricing changes, and policy interventions. Research demonstrates that the cumulative effect of such drivers is critical for achieving full adoption and reaching the maturity stage of a product's lifecycle (Kumar et al., 2020; Oliinyk et al., 2018; Bass et al., 1994). Policy-driven models, especially in high-investment or regulated sectors, show that government incentives and subsidies can accelerate adoption and stabilize adoption patterns, as seen in cross-country diffusion studies of renewable energy technologies (Guidolin et al., 2010). Similarly, marketing mix extensions that integrate advertising, price, distribution channels, and information flows – including word-of-mouth, online reviews, and search trends – have been shown to improve forecast accuracy and better reflect real-world adoption dynamics (Ramírez-Solís et al., 2022; Takahashi et al., 2024).

To capture non-linearities in diffusion, researchers have also introduced shock functions and change-point approaches. Min-Hi (2006) modeled abrupt changes in adoption rates by incorporating shocks affecting market potential, though assuming constant intensity for tractability. Guseo, Dalla Valle, and Guidolin (2007) applied exponential shock functions in the context of world oil depletion, demonstrating the adaptability of diffusion models beyond marketing. Kapur et al. (2007) introduced change-point models, where structural breaks modify model parameters across time intervals, and such approaches have since been applied in reliability engineering, software, and social media analytics (Huang et al., 2011; Irshad et al., 2019). Building on this line, Schweidel and Fader (2009) proposed a two-state adoption model transitioning from initial exponential purchases to an Erlang-2 distribution, improving sales forecasts, while Yu and Tseng (2015) developed a fuzzy piecewise logistic growth model that identifies significant change-points and provides confidence intervals for predictions. Parallel work in reliability engineering, such as the load-sharing and bathtub-shaped fault removal models by Gurov and Utkin (2012, 2014) and Peng, Liu, and Wang (2016), further illustrates the utility of piecewise functions to model complex, real-world systems. Further, stochastic and piecewise approaches have been extended to capture consumer behavior and technological substitution, accommodating external shocks like policy changes or competitor actions (Niu, 2006; Kapur et al., 2019; Panwar et al., 2019).

The reviewed literature demonstrates significant progress in extending the Bass model through marketing variables, shock functions, and change-point modeling, as well as through parallels drawn from reliability theory. However, existing approaches are often tailored to specific contexts and lack generalizability and produce statistically unstable estimates due to complex parameterizations. This study addresses these gaps by proposing a piecewise smooth innovation diffusion model that systematically incorporates random shifts due to internal and external factors. The framework unifies concepts from diffusion modeling and reliability engineering, while explicitly accommodating PLC deviations. Stability analysis ensures robustness, and empirical validation on multiple music technology datasets (cassette, compact disc, and video formats) demonstrates both practical applicability and statistical improvements over traditional Bass-based approaches.

3 Modelling Framework

Notation

m	Total market size
p	Innovation effect (external influence i.e., mass media)
q	Imitation effect (internal influence i.e., word-of-mouth)
$N(t)$	The cumulative number of adopters by time *t*
$w(t)$	Modulation function reflecting time varying external conditions.
$W(t)$	Cumulative modulation function representing the integrated effect of $w(t)$ over time.
$\mathrm{b_i}$	Piecewise adoption intensity parameter for the *i*-th interval
t_i	Change points / boundaries of piecewise intervals
T	Total time horizon
$H(t-t_i)$	Heaviside (step) function

3.1 Modelling Framework

Bass model (1969) is broadly utilized to forecast adoption of high technological products among the consumers. The model is based on the assumption that the total number of adopters by any time t is proportional to the total number of non-adopters left in the population. Further, the conversion rate from the non-adopters to adopters is assumed to be a pressure function based on the theory of innovators and imitators (Lee, Trimi and Kim, 2013). The corresponding differential equation is given by:

$$\frac{dN(t)}{dt} = \left[p + q\frac{N(t)}{m}\right][m - N(t)] \tag{1}$$

The coefficient of innovation *p* reflects the probability of adoption due to external influences such as mass media or public campaigns, while the coefficient of imitation *q* captures adoption driven by internal social contagion (word-of-mouth). As clarified by Mahajan, Muller & Bass (1990) and Van den Bulte & Stremersch (2004), *p* is associated with external drivers that act independently of prior adopters, whereas *q* depends on the proportion of existing adopters and hence represents internal influence. Therefore, *p* and *q* are not counts of adopters or non-adopters, but parameters that regulate the flow of adoption from the potential adopter pool $m - N(t)$.

The standard Bass diffusion model assumes a constant environment, with adoption dynamics fully characterized by the innovation p and q coefficients. However, real-world diffusion processes are often subject to sudden environmental shifts – for example, policy interventions, technological disruptions, or supply-side shocks – that invalidate the constant-environment assumption. In such cases, adoption dynamics evolve in *regime-dependent* ways. Accordingly, the coefficients of innovation p and imitation q are defined as time-varying functions modulated by $w(t)$, a function that encapsulates the temporal influence of environmental factors on the diffusion dynamics. Hence, coefficients of innovation and imitation takes the form:

$$\begin{aligned} p(t) &= p.w(t) \\ q(t) &= q.w(t) \end{aligned} \tag{2}$$

where $w(t) > 0$ for $t > 0$.

Using Equation (2), Equation (1) can be transformed into:

$$\frac{dN(t)}{dt} = \left[p + q\frac{N(t)}{m}\right][m - N(t)]w(t) \tag{3}$$

where $N(t)$ is the cumulative adoption fraction and $w(t)$ encodes environmental perturbations. To explicitly capture these structural breaks, $w(t)$ is modelled as a piecewise modulation function that scales the adoption intensity over time. Instead of treating shocks as random noise they are represented as structural changes in adoption regimes. Formally, the time horizon

$(0, T]$ is partitioned into sub-intervals $(t_{i-1}, t_i]$ where $t_0 = 0$ and $t_n = T$. Within each regime, $w(t)$ follows a parametric form (e.g., exponential decay, uniform constant stress), while across regimes it may shift discontinuously at change points $\{t_i\}$. This construction parallels classical change-point models in econometrics (Bai & Perron, 2003), ensuring theoretical legitimacy.

$$w(t) = \begin{cases} w_1(t); & t_0 < t \le t_1 \\ w_2(t); & t_1 < t \le t_2 \\ w_3(t); & t_2 < t \le t_3 \\ \quad \vdots; & \\ w_n(t); & t_{n-1} < t \le t_n \end{cases} \tag{4}$$

Solving the Equation (3) with initial conditions $N(0) = 0$, a generalized form for the cumulative number of adopters at any time t under regime specific changes can be obtained as follows:

$$N(t) = m\left[\frac{1 - exp\{-(p+q)W(t)\}}{1 + \frac{q}{p}exp\{-(p+q)W(t)\}}\right] \tag{5}$$

To preserve analytical tractability and avoid discontinuities in the diffusion trajectory, the cumulative modulation function $W(t)$ is constructed as a piecewise smooth function. This ensures that regime shifts are accommodated while maintaining global continuity of the adoption curve. Formally,

$$W(t) = \begin{cases} W_1(t) = \int_{t_0}^{t} w_1(x)dx; & t_0 < t \le t_1 \\ W_2(t) = W_1(t_1) + \int_{t_1}^{t} w_2(x)dx; & t_1 < t \le t_2 \\ W_3(t) = W_2(t_2) + \int_{t_2}^{t} w_3(x)dx; & t_2 < t \le t_3 \\ \quad \vdots; & \\ W_n(t) = W_{n-1}(t_{n-1}) + \int_{t_{n-1}}^{t} w_n(x)dx; & t_{n-1} < t \le t_n \end{cases} \tag{6}$$

Since the adoption dynamics are influenced by regime-dependent environmental shifts, the cumulative modulation function can be formulated as a piecewise function. Each regime is represented by a generalized Heaviside (step) function:

$$H(t - t_i) = \begin{cases} 1; & t \leq t_i \\ 0; & t > t_i \end{cases} \tag{7}$$

Within this framework, the cumulative modulation function $W(t)$ can be expressed as a sum over all sub-intervals $(t_{i-1}, t_i]$, such that

$$\begin{aligned} W(t) = {} & W_1(t)H(t - t_0)(1 - H(t - t_1)) \\ & + W_2(t)H(t - t_1)(1 - H(t - t_2)) \\ & + W_3(t)H(t - t_2)(1 - H(t - t_3)) + \cdots \\ & + W_n(t)H(t - t_{n-1})(1 - H(t - t_n)) \end{aligned} \tag{8}$$

Or

$$W(t) = \sum_{i=1}^{n} W_i(t)H(t - t_{i-1})(1 - H(t - t_i)) \tag{9}$$

where $W_i(t)$ defines the adoption intensity within the *i*-th regime, and the Heaviside functions ensure that shifts between regimes occur precisely at the change points $\{t_i\}$. This construction allows the model to capture both gradual and abrupt variations in adoption dynamics induced by environmental factors.

Therefore, the final expression for the cumulative adoption of innovation is written as:

$$N(t) = m\left[\frac{1 - exp\{-(p+q)\sum_{i=1}^{n} W_i(t)H(t - t_{i-1})(1 - H(t - t_i))\}}{1 + \frac{q}{p}exp\{-(p+q)\sum_{i=1}^{n} W_i(t)H(t - t_{i-1})(1 - H(t - t_i))\}}\right] \tag{10}$$

where *p* and *q* are the baseline innovation and imitation coefficients, respectively. This formulation preserves the piecewise structure of environmental influences while providing a mathematically tractable expression suitable for both analytical and numerical evaluation, enabling precise modeling of multi-peak, non-monotonic adoption patterns.

3.2 Parameter Bounds and Normalization Assumptions

In modeling adoption with piecewise extensions of the Bass framework, it is necessary to impose natural bounds on the parameters and modulation function to ensure interpretability and statistical stability.

The modulation function $w(t)$, which redistributes adoption intensity across phases, is constrained to lie within the normalized range $[0, 1]$. This ensures that $w(t)$ behaves as a probability-like weight rather than an uncontrolled scaling factor. Mapping each segment to a fraction of progress $[0, 1]$, or equivalently representing $w(t)$ as a probability density function (e.g., exponential kernel), guarantees that the contribution of each segment is bounded and interpretable.

Allowing $w(t)$ to exceed unity or vary arbitrarily introduces redundancy: multiple combinations of $(p, q, m, w(t))$ can replicate the same adoption curve. This redundancy leads to strong parameter correlations, inflated confidence intervals, and unstable estimates. By bounding $w(t)$, unnecessary degrees of freedom are removed, overfitting to random fluctuations is prevented, and statistically efficient estimates of the diffusion parameters are obtained. The coefficients p and q are restricted to be strictly positive $(p, q > 0)$. Here, p captures the innovation effect (spontaneous adoption due to advertising, external push, or curiosity), while q captures the imitation effect (social contagion and word-of-mouth). When multiplied by $w(t)$, both innovation and imitation intensities vary across time segments, allowing the model to flexibly accommodate real-world shocks and rejuvenations. Negative or unbounded values would contradict the behavioural interpretation and destabilize the adoption process. The market size parameter m is assumed to be finite and strictly positive $(m > 0)$. This prevents unbounded growth and ensures that cumulative adoption is limited by realistic demand conditions.

3.3 Different Forms of $w(t)$

For illustration of the working formula (10), two exclusive scenarios have been modeled below:

Model 1: Constant Piecewise Modulation Function

A constant piecewise modulation function is defined as a function that takes constant values over specified intervals of time. It assumes that the modulation effect remains constant within specified intervals but changes abruptly at the boundaries of these intervals. This type of function is particularly useful for modeling scenarios where external influences remain stable for certain periods before abruptly changing. For Example: A government introduces periodic incentives/subsidies or a product experiences demand surges during regulatory or seasonal shocks or Marketing Campaigns – A company runs distinct marketing campaigns during specific periods. The constant modulation function is given by:

$$w_i(t) = \frac{1}{t_i - t_{i-1}} \quad \forall\, t \in (t_{i-1}, t_i]; \quad i = 1, 2, 3 \ldots n \tag{11}$$

Firstly, assume there occur one turning or change point t_1 between the time horizon $(0, T]$ of the diffusion process, during which the adoption rate either increases or decreases. In such environment, the total adoption of the innovation by time *t* is given by:

$$N(t) = \begin{cases} m\left[\dfrac{1 - e^{-(p+q)\left(\frac{t}{t_1}\right)}}{1 + \frac{q}{p}e^{-(p+q)\left(\frac{t}{t_1}\right)}}\right]; & t \le t_1 \\ m\left[\dfrac{1 - e^{-(p+q)\left(1+\frac{t-t_1}{T-t_1}\right)}}{1 + \frac{q}{p}e^{-(p+q)\left(1+\frac{t-t_1}{T-t_1}\right)}}\right]; & t > t_1 \end{cases} \tag{12}$$

Suppose, there occur multiple break points 'n' during which the adoption rate changes at time points $t_1, t_2, \ldots, t_n$ respectively. The cumulative adoption of an innovation at time *t* is given by:

$$N(t) = \begin{cases} m\left[\dfrac{1 - e^{-(p+q)\left(\frac{t}{t_1}\right)}}{1 + \frac{q}{p}e^{-(p+q)\left(\frac{t}{t_1}\right)}}\right]; & 0 < t \le t_1 \\ m\left[\dfrac{1 - e^{-(p+q)\left(1+\frac{t-t_1}{t_2-t_1}\right)}}{1 + \frac{q}{p}e^{-(p+q)\left(1+\frac{t-t_1}{t_2-t_1}\right)}}\right]; & t_1 < t \le t_2 \\ m\left[\frac{1-e^{-(p+q)\left(2+\frac{t-t_2}{t_3-t_2}\right)}}{1+\frac{q}{p}e^{-(p+q)\left(2+\frac{t-t_2}{t_3-t_2}\right)}}\right]; & t_2 < t \le t_3 \\ .; & \\ .; & \\ .; & \\ m\left[\dfrac{1 - e^{-(p+q)\left(n+\frac{t-t_n}{\mathrm{T}-t_n}\right)}}{1 + \frac{q}{p}e^{-(p+q)\left(n+\frac{t-t_n}{\mathrm{T}-t_n}\right)}}\right]; & t_n < t \le T \end{cases} \tag{13}$$

Model 2: Continuous Piecewise Modulation Function

A continuous piecewise smooth modulation function is defined as a function that is continuous within specified intervals but may have different functional forms across these intervals. By being piecewise continuous, it allows for different growth or decay rates in different intervals of time, reflecting changes in external conditions or interventions. This type of function is suitable for modeling scenarios where external conditions change gradually over time. Hence, it is assumed that the modulation function follows an exponential distribution i.e., $w_i(t) = b_i e^{-b_i(t-t_{i-1})}$. Of late, this distribution has found to be suitable to model the economic or technological shocks inhibit in the underlying process.

Firstly, assume there occur only one turning point t_1 during which the adoption rate changes due to external environment. Then, the cumulative adoption function takes the form:

$$N(t) = \begin{cases} m\left[\dfrac{1-e^{-(p+q)(1-e^{-b_1 t})}}{1+\frac{q}{p}e^{-(p+q)(1-e^{-b_1 t})}}\right]; & t \leq t_1 \\ m\left[\dfrac{1-e^{-(p+q)((1-e^{-b_1 t_1})+(1-e^{-b_2(t-t_1)}))}}{1+\frac{q}{p}e^{-(p+q)((1-e^{-b_1 t_1})+(1-e^{-b_2(t-t_1)}))}}\right]; & t > t_1 \end{cases} \tag{14}$$

Further, for the case when there occur more than one change points, let's say, the adoption rate changes '*n*' times during the diffusion process at time points $t_1, t_2, \ldots, t_n$ respectively. Then, the cumulative adoption function takes the form:

$$N(t) = \begin{cases} m\left[\dfrac{1-e^{-(p+q)(1-e^{-b_1 t})}}{1+\frac{q}{p}e^{-(p+q)(1-e^{-b_1 t})}}\right]; & 0 < t \leq t_1 \\ m\left[\dfrac{1-e^{-(p+q)((1-e^{-b_1 t_1})+(1-e^{-b_2(t-t_1)}))}}{1+\frac{q}{p}e^{-(p+q)((1-e^{-b_1 t_1})+(1-e^{-b_2(t-t_1)}))}}\right]; & t_1 < t \leq t_2 \\ m\left[\dfrac{1-e^{-(p+q)((1-e^{-b_1 t_1})+(1-e^{-b_2(t_2-t_1)})+(1-e^{-b_3(t-t_2)}))}}{1+\frac{q}{p}e^{-(p+q)((1-e^{-b_1 t_1})+(1-e^{-b_2(t_2-t_1)})+(1-e^{-b_3(t-t_2)}))}}\right]; & t_2 < t \leq t_3 \\ .; & \\ .; & \\ .; & \\ m\left[\dfrac{1-e^{-(p+q)\left(\sum_{i=1}^{n}(1-e^{-b_i(t_i-t_{i-1})})+(1-e^{-b_{n+1}(t-t_n)})\right)}}{1+\frac{q}{p}e^{-(p+q)\left(\sum_{i=1}^{n}(1-e^{-b_i(t_i-t_{i-1})})+(1-e^{-b_{n+1}(t-t_n)})\right)}}\right]; & t_n < t \leq T \end{cases} \tag{15}$$

The above models has been discussed in the numerical illustration i.e., Section 4. In next sub section, stability analysis of the proposed approach has been done.

3.4 Stability Analysis

The stability analysis helps identify conditions under which adoption begins or stalls and provides guidance on strategies to influence these conditions effectively. Understanding these dynamics allows marketers, policymakers, and businesses to tailor their approaches to promote or sustain the adoption of new products and technologies.

To find the equilibrium points, the rate of change $\frac{dN(t)}{dt}$ is set to be zero:

$$\frac{dN(t)}{dt} = \left[p + q\frac{N(t)}{m}\right][m - N(t)]w(t) = 0 \tag{16}$$

This results in potential equilibrium points at $N = 0$ and $N = m$ when $w(t) \neq 0$. To analyse the stability of these equilibrium points, linearization around each equilibrium point is performed:

$$\frac{dN(t)}{dt} = f(N) = \left[p + q\frac{N(t)}{m}\right][m - N(t)]w(t) \tag{17}$$

The general approach to linearization involves computing the derivative of the right-hand side of the differential equation with respect to N:

$$f'(N) = w(t)\left(q - p - \frac{2qN}{m}\right) \tag{18}$$

Evaluating at Equilibrium Points:

1. **At $N = 0$:**

$$f'(0) = w(t)(q - p)$$

The stability condition depends on the sign of $q - p$:

- If $q - p > 0$, then $f'(0) > 0$, indicating that $N = 0$ is unstable. The system is likely to move away from zero adoption. This could be due to strong social influences (high q) compared to the intrinsic value of the innovation (lower p). Initial adopters quickly influence others, leading to a rapid increase in adoption.
- If $q - p < 0$, then $f'(0) < 0$, indicating that $N = 0$ is stable.

Initial adoption is slow or non-existent, potentially due to high initial costs, lack of awareness, or insufficient intrinsic value. There is less likelihood of adoption spreading purely by imitation.

2. **At $N = m$:**

$$f'(N) = w(t)(-p - q)$$

- Since p and q are typically positive, $-p - q < 0$, indicating that $N = m$ is stable. This stability suggests that once the product reaches saturation, the market will not easily revert to a non-adoption state, indicating strong product retention and sustained market presence..

This detailed stability analysis shows that the equilibrium points $N = 0$ and $N = m$ have different stability characteristics depending on the parameters p and q, with physically reasonable nonnegative p, q and positive w, $N = 0$ is typically unstable (so adoption will start) and $N = m$ is globally attractive (the market saturates).

4 Numerical Illustration

To validate the model on real-world data, historical datasets from the US recorded music industry – covering sales volumes across different formats – were utilized. These datasets span complete product life cycles, capturing the full trajectory of adoption and diffusion. Through the history of music industry, two most useful music record formats before non-physical format (download music) can be acknowledged. These were: Cassettes, Compact Disc. After that, there came video concept of music; the sales of video formats in comparison of audio were not much high but requires a special place in the evolution of music industry. The data of these recorded music sales have been fetched from RIAA (U.S. Music Revenue Database – RIAA, 2023). The first dataset (DS-1) corresponds to the cassette sales (in million units), second database (DS-2) corresponds to the compact disk database and the third dataset (DS-3) corresponds to the sales of physical music records of music videos (in million units). The respective PLCs of these datasets give an idea for how long the respective technology remains relevant through the continued use of it. Hence these datasets are selected. It is important to note that while the Bass model was originally developed for durable goods, prior literature has successfully applied it to music formats (Boswijk & Franses, 2005; Guseo & Guidolin, 2015).

The number of change points was determined using a balance between model fit and parsimony. However, in order to prevent overfitting and preserve interpretability, the maximum number of change points are restricted to two for DS-2 and 3, and three for DS-1. This choice is consistent with the observed changes in the adoption curve and corresponds to meaningful regime shifts. Models with additional breakpoints were considered, but those models only yield marginal improvements in fit at the cost of increased complexity. The final specification thus balances parsimony with explanatory power.

4.1 Parameter Estimation

The parameters of the proposed models were estimated using non-linear least squares fitting on three datasets: cassette sales, compact disc sales, and music video sales. The parameters are reported in Tables 1–3.

4.1.1 Cassettes

Model 1 (Uniform, piecewise smooth function) identified three change points at approximately periods 5.1, 15.1, and 26.6. The estimated market potential was $m = 6316.6$, with a very low baseline innovation-driven adoption intensity ($p = 0.003$) and a moderate baseline imitation intensity ($q = 3.188$). This specification does not include regime-specific parameters, implying that adoption evolves in a relatively uniform manner across phases. While parsimonious, this model smooths over sharper transitions and captures only the broad outline of the adoption curve.

Model 2 (Exponential, piecewise smooth function) imposed breakpoints later in the series, at around periods 12.8, 21.0, and 25.5. The market potential was slightly lower ($m = 6202.4$), but the dynamics were much stronger, with a higher baseline innovation parameter ($p = 0.040$) and a very large baseline

Table 1 Parameter estimates of cassette dataset

Cassette			Parameter									
			m	p	q	b_1	b_2	b_3	b_4	t_1	t_2	t_3
Model 1	Estimate		6316.617	0.003	3.188	–	–	–	–	5.095	15.066	26.59
	Std. Error		32.262	0.002	0.271	–	–	–	–	0.887	0.593	1.115
	95% C.I.	L.B.	6250.729	6.9E-05	2.635	–	–	–	–	3.283	13.854	24.312
		U.B.	6382.505	0.006	3.742	–	–	–	–	6.906	16.278	28.868
Model 2	Estimate		6202.395	0.040	17.376	0.026	0.017	0.020	0.033	12.771	20.951	25.461
	Std. Error		6.571	0.016	4.070	0.008	0.004	0.005	0.009	0.625	0.163	0.226
	95% C.I.	L.B.	6188.889	0.006	9.011	0.009	0.008	0.010	0.016	11.486	20.615	24.997
		U.B.	6215.902	0.073	25.742	0.043	0.026	0.030	0.051	14.056	21.287	25.925

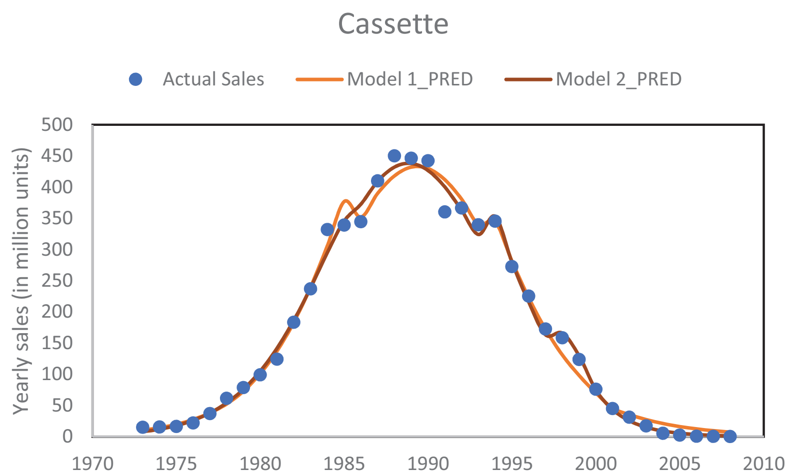

Figure 1 Goodness of fit curve for DS-1.

imitation effect ($q = 17.376$). Importantly, this model estimated regime-specific adjustment parameters ($b_1 - b_4$), reflecting different exponential growth dynamics across phases. For instance, growth accelerated sharply in the early and late stages, with a temporary slowdown in the middle stages. This richer structure provides a closer alignment with the Bass diffusion framework, in which adoption accelerates through social contagion before levelling off. The actual and predicted values of both the models are encapsulated in Figure 1. It is clear from the graph that Model 2 aligns closely to the actual dataset.

4.1.2 Compact disc

Model 1 estimated a market potential of $m = 14,900.1$, with a baseline innovation parameter $p = 0.017$ and a relatively modest baseline imitation parameter ($q = 3.509$). Breakpoints were identified at 10.6 and 24.8, dividing the sales trajectory into three phases of adoption. The standard errors for the parameters were small, and the 95% confidence intervals were fairly narrow, for example, the market potential lies between 14,792–15,008. This indicates that the estimates are statistically reliable, though the model imposes uniform dynamics across regimes and does not capture phase-specific accelerations.

Model 2 yielded a slightly higher market potential of $m = 15,078.4$. The baseline innovation parameter was lower ($p = 0.008$), but the baseline imitation effect was considerably stronger ($q = 9.793$), suggesting that adoption was more heavily driven by peer influence. Breakpoints were

Table 2 Parameter estimates of CD dataset

Compact Disk			Parameter							
			m	p	q	b_1	b_2	b_3	t_1	t_2
Model 1	Estimate		14900.13	0.017	3.509	–	–	–	10.639	24.831
	Std. Error		53.249	0.005	0.149	–	–	–	0.719	0.642
	95% C.I.	L.B.	14792.029	0.008	3.206	–	–	–	9.179	23.528
		U.B.	15008.231	0.027	3.812	–	–	–	12.099	26.134
Model 2	Estimate		15078.443	0.008	9.793	0.071	0.026	0.025	15.446	20.988
	Std. Error		19.062	0.002	0.160	0.006	0.001	0.001	0.502	0.498
	95% C.I.	L.B.	15039.615	0.004	9.467	0.059	0.025	0.024	14.422	19.974
		U.B.	15117.271	0.012	10.119	0.084	0.028	0.026	16.469	22.002

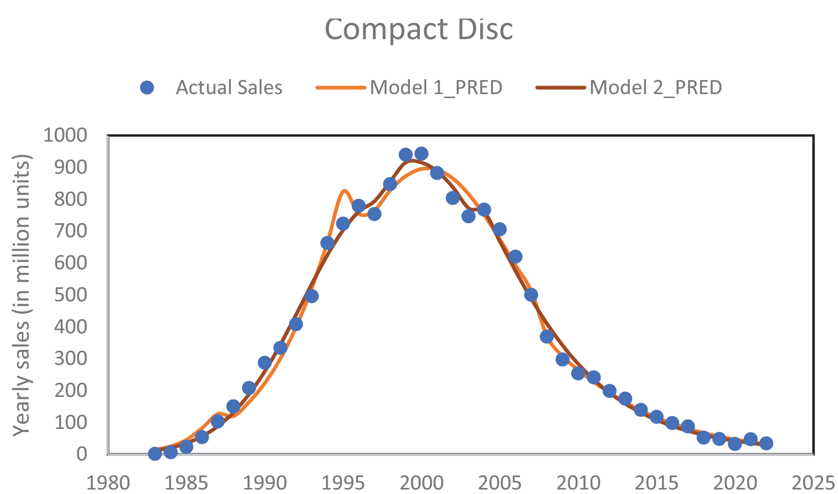

Figure 2 Goodness of fit curve for DS-2.

identified earlier, at 15.4 and 21.0, pointing to sharper regime shifts compared to Model 1. In addition, regime-specific exponential adjustments were estimated, reflecting a period of rapid initial acceleration followed by steadier growth. Standard errors were very small, and the 95% confidence intervals were extremely tight (e.g., m between 15,039–15,117), confirming the precision of the estimates. The actual and predicted values of both the models are encapsulated in Figure 2. It is clear from the graph that Model 2 aligns closely to the actual values.

4.1.3 Video records

Model 1 estimated a market potential of $m = 402.9$, with a potential innovation intensity of $p = 0.208$ and a modest baseline imitation effect $(q = 2.69)$.

Table 3 Parameter estimates of physical video dataset

Video Records			Parameter							
			m	p	q	b_1	b_2	b_3	t_1	t_2
Model 1	Estimate		402.919	0.208	2.69	–	–	–	15.017	24.393
	Std. Error		1.375	0.009	0.128	–	–	–	0.339	0.525
	95% C.I.	L.B.	400.111	0.19	2.428	–	–	–	14.324	23.321
		U.B.	405.726	0.226	2.952	–	–	–	15.71	25.464
Model 2	Estimate		418.336	0.722	17.325	0.014	0.010	0.103	11.323	15.252
	Std. Error		14.393	0.257	5.002	0.005	0.004	0.046	0.416	0.161
	95% C.I.	L.B.	388.805	0.196	7.062	0.004	0.002	0.008	10.470	14.922
		U.B.	447.867	1.249	27.587	0.025	0.018	0.198	12.176	15.582

Breakpoints were located at 15 and 24.4. The standard errors are very small (e.g., $SE = 1.375$ for m), and the 95% confidence intervals are narrow (e.g., m between 400.1–405.7), indicating precise estimates. This model suggests a relatively smooth and gradual diffusion process, with moderate innovation and limited peer-driven effects.

Model 2 estimated a higher market potential $(m = 418.3)$, along with a substantially larger innovation potential $(p = 0.722)$ and a much stronger imitation intensity $(q = 17.325)$. Regime-specific exponential adjustments were included, indicating heterogeneous adoption dynamics across phases, with a particularly sharp acceleration captured in the final regime. Breakpoints were identified earlier, at 11.3 and 15.3, reflecting more abrupt shifts in adoption speed than Model 1. Standard errors are somewhat larger, particularly for q (SE = 5.002), and the 95% confidence intervals are wider (e.g., m between 388.8–447.9), reflecting greater variability in parameter estimates.

The actual and predicted values of both the models are encapsulated in Figure 3. It is clear from the graph that Model 2 aligns closely to the actual dataset.

4.2 Model Comparison

The performance of the proposed piecewise smooth models was compared against the change point model which has application in both marketing & reliability contexts (Kapur et al. 2007) and traditional Bass diffusion model across the three datasets (Cassette, Compact Disc, and Video Records). Model fit was evaluated using four metrics: coefficient of determination (R^2), mean squared error (MSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). R^2 measures explained variance, MSE reflects average squared error, while AIC and BIC assess model fit penalized

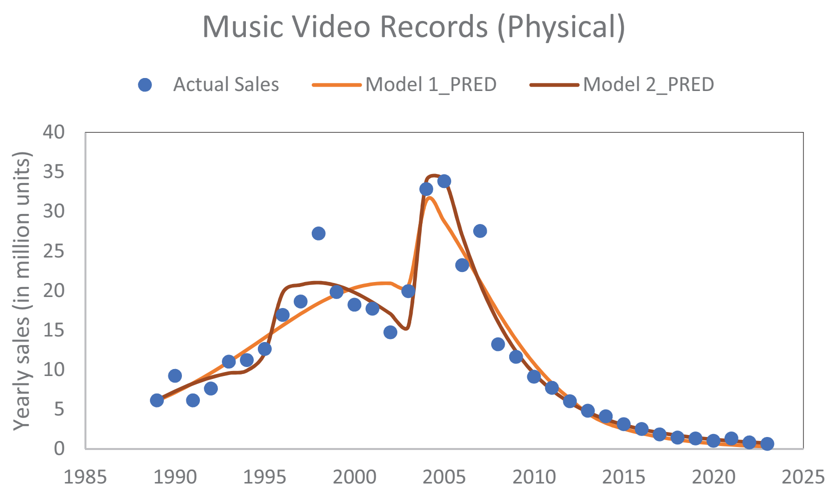

Figure 3 Goodness of Fit curve for DS-3.

for complexity – lower values indicate better models. Also, this residual analysis has been done on the non-cumulative predicted values of sales datasets.

- DS-1 (Table 4): The traditional Bass model performs weakest, with the lowest R^2, highest error, and least favourable information criteria, indicating its limited ability to capture structural shifts in adoption trajectories. The change-point model of Kapur et al. (2007) offers improved fit over Bass. However, its AIC and BIC values remain relatively high, suggesting that the improvement in explanatory power comes at the cost of additional complexity. Both proposed piecewise smooth models outperform the benchmarks. Model 2, however, demonstrates the best overall performance.
- DS-2 (Table 5): The proposed piecewise smooth models again outperform the benchmarks. Model 1 achieves a comparable fit but with higher MSE, although it produces more favourable AIC/BIC values than Bass and Kapur. Model 2 is the most robust performer, with the best overall explanatory power, the lowest error, and the most favourable AIC/BIC values.
- DS-3 (Table 6): While the overall explanatory power was lower compared to the other datasets, Model 2 remained the strongest performer, followed by Model 1 with the Kapur et al. (2007) model trailing significantly. The AIC and BIC values also consistently favoured Model 2.

Table 4 Model performances on cassette dataset (DS-1)

n.	R^2	MSE	AIC	BIC
Proposed Model 1	0.9891	265.1827	212.90	222.40
Proposed Model 2	0.9935	156.5951	201.93	217.77
Kapur et al. (2007)	0.9910	218.0155	217.84	236.85
Bass (1969)	0.9772	554.9676	233.48	238.23

Table 5 Model performances on compact disc dataset (DS-2)

n.	R^2	MSE	AIC	BIC
Proposed Model 1	0.9895	1034.1059	287.65	296.1
Proposed Model 2	0.9946	526.3918	266.64	280.15
Kapur et al. (2007)	0.9906	918.2454	290.90	306.10
Bass (1969)	0.9801	1967.4216	309.38	314.45

Table 6 Model performances on video records dataset (DS-3)

n.	R^2	MSE	AIC	BIC
Proposed Model 1	0.9175	7.1787	81.50	88.26
Proposed Model 2	0.9467	4.6379	72.03	85.54
Kapur et al. (2007)	0.9331	5.8043	83.00	98.20
Bass (1969)	0.7878	18.4136	117.18	122.25

Across all datasets, the proposed piecewise smooth models consistently outperformed the traditional Bass diffusion model and the change-point framework of Kapur et al. (2007). While the Bass model failed to capture turning points in adoption dynamics, the change-point model provided moderate improvement but at the cost of higher complexity and less favourable information criteria. By contrast, exponential-based modulation with regime-specific adjustments (Proposed Model 2) offered the best balance of fit, predictive accuracy, and parsimony, demonstrating that smooth modulation of diffusion parameters is more effective than imposing abrupt structural breaks. This confirms that allowing for regime-specific exponential adjustments provides a more accurate and theoretically consistent representation of product adoption dynamics than either uniform piecewise models or the classical Bass specification.

5 Discussion

Real-world diffusion processes is shaped by a dynamic marketing environment, where internal factors (such as pricing, promotions, and distribution)

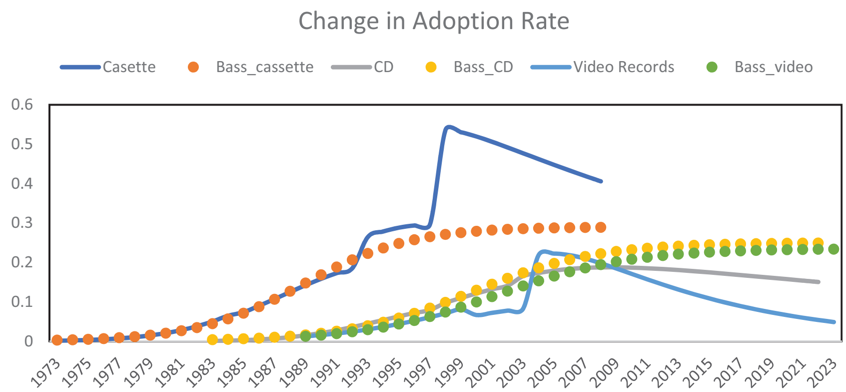

Figure 4 Impact of piecewise function on adoption rate.

and external factors (such as policy changes, technological disruptions, or competitive launches) alter the pace of adoption in unpredictable ways. To address this, the present study developed an extended diffusion framework that incorporates a modulation function $w(t)$, allowing the adoption rate to vary across phases. By testing both constant and continuous piecewise functions, the model offers greater flexibility and accuracy in representing real-world scenarios where abrupt shifts or gradual transitions are observed.

Empirical results from U.S. music record sales support this framework. As shown in Figure 4, the adoption rate initially mirrors the Bass model, but subsequently deviates, with distinct jumps that the classical model fails to capture. For cassettes, sharp increases occurred around 1992 and 1997; for compact discs, adoption accelerated in 1995 and again in 2002; and for music videos, sales declined in 1992 before rebounding in 2002. These turning points align with known market events and demonstrate that the continuous exponential piecewise function is especially effective in capturing complex adoption dynamics. Unlike the standard model, which smooths over such irregularities, the proposed framework highlights the importance of modeling adoption as a sequence of phase-specific processes influenced by external factors.

Beyond theoretical contributions, the findings have practical implications for managers and policymakers. The piecewise framework shows that adoption trajectories are rarely smooth, and that abrupt accelerations or declines are often triggered by external shocks (policy incentives, technological disruptions, promotions, product design, supply chain failures). By identifying these change points, managers can better align strategy with

Table 7 Managerial implications of the piecewise diffusion model

Model Insight	Managerial Implication	Practical Strategy
Adoption curves exhibit abrupt changes rather than smooth S-shapes.	Firms must anticipate non-linear adoption dynamics.	Develop contingency plans for sudden slowdowns (boost promotions) or surges (scale supply/distribution).
Piecewise phases represent distinct adoption stages.	Marketing should be tailored to each phase.	Early: awareness campaigns; Mid: peer-to-peer referral and bundling; Late: rebranding, upgrades, or targeting niche users.
Modulation $\boldsymbol{w(t)}$ governs adoption intensity across time.	Resource allocation should synchronize with peak adoption phases.	Adjust ad spend, inventory, and distribution dynamically with predicted adoption surges.
External shocks (policy, competition, economic crises) strongly influence adoption.	Firms must integrate environmental scanning into strategic planning.	Track regulations, competitor moves, and macroeconomic conditions to anticipate shifts.

market dynamics. Table 7 summarizes the key managerial insights derived from the model and its practical applications.

Future research should focus on refining the piecewise functions and exploring alternative forms for $w(t)$. Additionally, investigating the model's applicability to other industries and incorporating real-time data could further enhance its accuracy and relevance. Further studies could also explore the integration of additional external factors, such as social media influence and economic indicators, to provide a more comprehensive understanding of the adoption process.

6 Conclusion

This study presented a piecewise smooth extension of the Bass diffusion model to capture random shifts in adoption trajectories. By integrating a modulation function, the framework accommodates both abrupt and gradual changes in market dynamics, offering a more realistic depiction of innovation diffusion. Applied to the music record industry, the model consistently identified turning points and replacement cycles, outperforming standard formulations in both fit and stability. These results underscore the importance of accounting for environmental fluctuations in diffusion studies and provide a foundation for more adaptive forecasting tools.

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