On Complete Diallel Cross Plans

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Abstract

A diallel cross is a pairing strategy commonly employed by animal husbandry scientists and botanists to explore the genetic interactions and inheritance patterns among a set of l inbred lines. A commonly used diallel cross design is a complete diallel cross (CDC) plan where each line is crossed with the remaining $(l-1)$ distinct lines, resulting in $l(l-1)/2$ number of crosses in the entire design.

In this investigation, we present a method for constructing complete diallel cross schemes based on balanced incomplete block design (BIBD) of series $t = b = l, r = k = l-1$, and $\lambda = l-2$. Here, crosses are made in such a way that one line will cross with other line within a block. Two methods viz, (i) binary complete diallel cross plan and (ii) non-binary CDC plans are developed from the same series of BIBD to suggest which plans should be accepted by the breeder for their experiment. The construction of CDC plan

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is demonstrated through appropriate examples. We compute the efficiency of the CDC plan and compare it with randomized complete block design. It is shown that the constructed CDC plan is universally optimal. Robustness of CDC plan is examined in relation to the loss of one block.

Keywords: BIB design, CDC plan, GCA, optimality, efficiency.

1 Introduction

A diallel cross is a mating strategy which was first proposed by Schmidth (1919). Since then, these schemes are widely employed by biologists in breeding experiments, enabling researchers to systematically explore the genetic interactions and inheritance patterns among a set of inbred lines. In particular, it is effective for estimating and comparing the genetic effects of combining abilities of the lines. To support the analysis of such schemes, suitable methods and models were developed by Griffing (1956).

Consider a diallel experiment in which there are l inbred lines $(\ell_1, \ell_2, \ldots, \ell_l)$, and a cross between the two lines ℓ_i and ℓ_j is denoted as (ℓ_i, ℓ_j) where $i < j = 1, 2, \ldots, l$. When every distinct pairwise crosses among these lines are taken together, the resulting plan is referred to as a complete diallel cross (CDC) IV plan. It leads to $l(l-1)/2$ distinct crosses in total. In the literature of diallel crosses, the performances of an individual parental line and a specific cross between two parental lines are respectively assessed by using the concept of general combining ability (GCA) and specific combining ability (SCA) effects. These concepts are central to genetic evaluation in breeding programs. The GCA effect reflects the average performance of a parental line when mated with other lines available in the experiment and it captures the additive genetic effects contributed by a parent and the SCA effect gives a measure of the performance of a specific cross, say (ℓ_i, ℓ_j), and captures the specific interaction between the ith and jth line effects.

In a CDC plan, total number of crosses grows at a quadratic rate as the number of lines, l, increases. For example, when the number of parental lines is $l = 4$, there are just six possible crosses, whereas with $l = 11$ the number of crosses grows significantly to fifty-five. In this scenario, applying a randomized complete block design (RCBD) with each cross treated as a treatment may become impractical as large number of treatments per block may cause error variance to become large. Therefore, it is preferable to employ a suitable incomplete block design (IBD) instead of a complete block design. Balanced incomplete block designs (BIBD), originally introduced by Yates (1936) and further developed by Bose (1939), have been widely utilized in the construction of CDC plans as they provide precise estimates of general combining ability effects. Numerous researchers, including Divecha and Ghosh (1994), Dey (2002), Das and Ghosh (1999), among others, discussed the methods of construction of CDC plans. It is established that connected and binary CDC plan is always optimal. Divecha and Ghosh (1994) obtained some incomplete block designs for diallel crosses. Das and Ghosh (1999) obtained some CDC plans. Later, Ghosh and Biswas (2003) utilized Galois fields to construct CDC plan using Galois field and showed the CDC plans constructed through two BIBDs with the identical parameters are universally optimal. More recently, Singh and Sharma (2022) obtained CDC plan using Galois field. Shaimaa et al. (2022) applied diallel cross plans to investigate the response of grain yield.

In this article, we develop some CDC plans using BIBD of series $t = b = l$, $r = k = l - 1$, and $\lambda = l - 2$, where $l > 2$ is an odd number which represents the number of lines, l denotes number of lines, b denotes blocks, k block size, r denotes replication size and λ denotes the number of times a pair of lines occur together in the design. Also, we derived the efficiency factor of CDC plans as well as the BIBD through which CDC plan is constructed. Using these efficiency factors and the estimates of the line effects, we can suggest to the plant breeders which plan will suit best for their experiment.

2 Estimation of GCA Effects

Let us assume a complete diallel cross experiment with l distinct parental lines. The resulting $\frac{l(l-1)}{2}$ non-reciprocal crosses are allocated among n experimental units, which are organized across b blocks, each containing k units. Suppose that a block design $d \in \mathcal{D}(t, b, k)$ is employed to conduct the experiment, where t represents the total number of treatments. Then, the statistical model to describe the observed response vector $\boldsymbol{y} \in \mathbb{R}^{n\times 1}$ is given as

$$\boldsymbol{y} = \mu \mathbf{1}_n + \boldsymbol{Z}_1\boldsymbol{\tau} + \boldsymbol{Z}_2\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

where μ is mean, $\mathbf{1}_n$ is an $n \times 1$ vector whose elements are 1, $\boldsymbol{\tau} \in \mathbb{R}^{l\times 1}$ represents the GCA effects and $\boldsymbol{\beta} \in \mathbb{R}^{b\times 1}$ represents the block effects, $\boldsymbol{Z}_1 \in \mathbb{R}^{n\times l}$ and $\boldsymbol{Z}_2 \in \mathbb{R}^{n\times b}$ are design matrices associated with the lines and blocks respectively and $\boldsymbol{\varepsilon} \in \mathbb{R}^{n\times 1}$ is the vector of residual errors whose each element is distributed $N(0, \sigma^2)$. Here, $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2 \boldsymbol{I}_n$. These errors primarily account for specific combining ability and other unexplained sources of variation.

The entry at (i, j)th position of matrix $\boldsymbol{Z}_1$ is 1, if the ith observation involves jth parental line and 0 otherwise. Likewise, a value of 1 in matrix $\boldsymbol{Z}_2$ indicates that the observation falls within the jth block and 0 otherwise.

Following the formulation by Gupta and Kageyama (1994), the $\boldsymbol{\mathcal{C}}_d$ matrix for estimating the GCA effects is:

$$\boldsymbol{\mathcal{C}}_d = \boldsymbol{\mathcal{M}}_d - \boldsymbol{N}_d \boldsymbol{K}^{-1} \boldsymbol{N}_d' \tag{2}$$

where, $\boldsymbol{\mathcal{M}}_d = \boldsymbol{Z}_1' \boldsymbol{Z}_1$ is a symmetric matrix which indicates the frequency with which each line and each cross appears in the design, $\boldsymbol{N}_d = \boldsymbol{Z}_1' \boldsymbol{Z}_2$ records the number of times each line is observed within each block, $\boldsymbol{K} = \boldsymbol{Z}_2' \boldsymbol{Z}_2$ is a diagonal matrix containing block sizes. Here, we consider the case in which the blocks are of equal sizes and thus $\boldsymbol{K} = k\boldsymbol{I}_b$.

Then, the $\boldsymbol{\mathcal{C}}_d$ matrix simplifies to

$$\boldsymbol{\mathcal{C}}_d = \boldsymbol{\mathcal{M}}_d - k^{-1} \boldsymbol{N}_d \boldsymbol{N}_d'$$

The corresponding reduced normal equations for GCA effects are,

$$\boldsymbol{\mathcal{C}}_d \hat{\boldsymbol{\tau}} = T - k^{-1} \boldsymbol{N}_d B \tag{3}$$

where $T \in \mathbb{R}^{l \times 1}$ and $B \in \mathbb{R}^{b \times 1}$ denote the vectors of line and block totals respectively.

The estimates of GCA effects and the corresponding variances are given as

$$\hat{\tau}_i = \frac{1}{\varphi} Q_i, \quad i = 1, 2, \ldots, l. \tag{4}$$

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{\varphi} \sigma^2, \quad i \neq j = 1, 2, \ldots, l. \tag{5}$$

where $\hat{\tau}_i$ is the estimate of ith line effect, Q_i is the adjusted line total for ith line, $\varphi > 0$ is the non-zero eigen root of the $\boldsymbol{\mathcal{C}}_d$ matrix, $\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)$ denotes variance of $(\hat{\tau}_i - \hat{\tau}_j)$ and σ^2 is error variance. The matrix $\boldsymbol{\mathcal{M}}_d$ plays a key role in estimating the GCA effects as it reflects how frequently each line and each cross appears in the design. It can also be expressed as

$$\boldsymbol{\mathcal{M}}_d = \begin{pmatrix} m_{11} & m_{12} & \ldots & m_{1l} \\ m_{21} & m_{22} & \ldots & m_{2l} \\ \ldots & \ldots & \ldots & \vdots \\ m_{l1} & m_{l2} & \ldots & m_{ll} \end{pmatrix}$$

where, m_{ii} represents replication count of parent ℓ_i in the design. This means, it represents how many crosses include the line ℓ_i. The elements m_{ij} represent how many times the cross (ℓ_i, ℓ_j) is repeated in the plan.

3 Efficiency Factor of CDC Plan

Let us use a Randomized Complete Blocked Design (RCBD) for a diallel experiment instead of the CDC plan. Let the RCBD consists of r blocks, each containing $\frac{(l-2)}{2}$ crosses. In this set up, total number of crosses in the experiment is $\frac{r(l-2)}{2}$.

Under this design, $\mathcal{C}_d$-matrix of the RCBD is obtained from,

$$\mathcal{C}_d = r(l-2)\left[I_l - \frac{E_{ll}}{l}\right].$$

Now, under RCBD, variance of BLUE of GCA effects is defined as,

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{r(l-2)}\sigma^2, \quad i \neq j = 1, 2, \ldots, l;$$

where, $\hat{\tau}_i$ is the estimate of ith line effect, r denotes how many times a cross is repeated in CDC plan and σ^2 is error variance.

The efficiency (E) of the CDC plan relative to RCBD in r replications is then defined as

$$\mathrm{E} = \frac{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{RCBD}}{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{CDC}} = \frac{\frac{2}{r(l-2)}\sigma^2}{\frac{2}{\varphi}\sigma^2} = \frac{\varphi}{r(l-2)}.$$

4 Method of Construction

Select BIBD with $t = l = b, r = k = l - 1$, and $\lambda = l - 2$. The method of construction of BIBD with these parameters is available in the literature. We discuss the method of CDC plan construction using two techniques explained as Case I and Case II.

Case I: Binary complete diallel cross plan

Take a BIBD with $t = l = b, r = k = l - 1$, and $\lambda = l - 2$. Let l is an odd number, then r and k will be even number. Select one block from the given BIBD, take cross between the lines ℓ_i and ℓ_j in the same block such that $i < j$ and no line is repeated within the block. Thus, a block with $k = (l-1)$

lines will produce $\frac{(l-1)}{2}$ distinct crosses. Apply the same rule to the remaining $(l-1)$ blocks such that there is no repetition of any cross that has already occurred in the previous block(s). Since we have l blocks and hence the total crosses obtained in the plan is $\frac{l(l-1)}{2}$ which proves the constructed plan is complete diallel cross plan. Now we have a CDC plan where $\frac{l(l-1)}{2}$ crosses are arranged in $b = l$ blocks, where every line occurs $(l-1)$ times across all the blocks and each cross appears exactly once. Thus, the resulting design is binary.

The $\mathcal{C}_d$ matrix of the CDC plan is written as

$$\mathcal{C}_d = \begin{bmatrix} (l-1) & 1 & \dots & 1 & 1 \\ 1 & (l-1) & \dots & 1 & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & (l-1) & 1 \\ 1 & 1 & \dots & 1 & (l-1) \end{bmatrix}$$

$$- \begin{bmatrix} (l-1) & (l-2) & \dots & (l-2) & (l-2) \\ (l-2) & (l-1) & \dots & (l-2) & (l-2) \\ \dots & \dots & \dots & \dots & \dots \\ (l-2) & (l-2) & \dots & (l-1) & (l-2) \\ (l-2) & (l-2) & \dots & (l-2) & (l-1) \end{bmatrix} \Bigg/ \frac{(l-1)}{2}$$

$$= \begin{bmatrix} \frac{(l-1)(l-3)}{2} & -\frac{(l-3)}{2} & \dots & -\frac{(l-3)}{2} & -\frac{(l-3)}{2} \\ -\frac{(l-3)}{2} & \frac{(l-1)(l-3)}{2} & \dots & -\frac{(l-3)}{2} & -\frac{(l-3)}{2} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{(l-3)}{2} & -\frac{(l-3)}{2} & \dots & \frac{(l-1)(l-3)}{2} & -\frac{(l-3)}{2} \\ -\frac{(l-3)}{2} & -\frac{(l-3)}{2} & \dots & -\frac{(l-3)}{2} & \frac{(l-1)(l-3)}{2} \end{bmatrix} \Bigg/ \frac{(l-1)}{2}$$

$$\mathcal{C}_d = \frac{\frac{l(l-3)}{2}}{\frac{(l-1)}{2}} \left[I_l - \frac{1}{l} E_{ll} \right] = \frac{l(l-3)}{l-1} \left[I_l - \frac{1}{l} E_{ll} \right]$$

where, $\varphi = \frac{l(l-3)}{l-1} > 0$ is the eigen root of $\mathcal{C}_d$ matrix of the CDC plan.

4.1 Estimates and Efficiency Factor of the Binary CDC Plan

For the CDC plan described in Case I, the estimates of GCA effects and the variance of GCA effects under the CDC plan are given below.

The estimator of GCA effects is

$$\hat{\tau}_i = \frac{1}{\varphi} Q_i = \frac{1}{\frac{l(l-3)}{l-1}} Q_i = \frac{(l-1)}{l(l-3)} Q_i,$$

where, Q_i is the adjusted line total for ith line. The variance of GCA effects under CDC plan is given as

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{\frac{l(l-3)}{l-1}} \sigma^2 = \frac{2(l-1)}{l(l-3)} \sigma^2.$$

Similarly, variance under RCBD of GCA effects is defined as,

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{r(l-2)} \sigma^2, \quad \text{where } i \neq j = 1, 2, \ldots, l.$$

In this example, $r = 1$ because each cross is repeated only once. Thus,

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{(l-2)} \sigma^2.$$

Efficiency E follows as

$$\mathrm{E} = \frac{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{RCBD}}{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{CDC}} = \frac{\frac{2}{(l-2)} \sigma^2}{\frac{2(l-1)}{l(l-3)} \sigma^2} = \frac{l(l-3)}{(l-1)(l-2)}.$$

The complete procedure to obtain a binary CDC plan for $l = 7$ is illustrated in the example given below.

Example 4.1. Consider a BIBD with parameters $t = 7, b = 7, r = 6, k = 6,$ and $\lambda = 5$. The seven blocks of this design are as follows:

$$\begin{array}{cc}
(\ell_1 \;\; \ell_2 \;\; \ell_3 \;\; \ell_4 \;\; \ell_5 \;\; \ell_6), & (\ell_1 \;\; \ell_2 \;\; \ell_3 \;\; \ell_4 \;\; \ell_5 \;\; \ell_7), \\
(\ell_1 \;\; \ell_2 \;\; \ell_3 \;\; \ell_4 \;\; \ell_6 \;\; \ell_7), & (\ell_1 \;\; \ell_2 \;\; \ell_3 \;\; \ell_5 \;\; \ell_6 \;\; \ell_7), \\
(\ell_1 \;\; \ell_2 \;\; \ell_4 \;\; \ell_5 \;\; \ell_6 \;\; \ell_7), & (\ell_1 \;\; \ell_3 \;\; \ell_4 \;\; \ell_5 \;\; \ell_6 \;\; \ell_7), \\
\multicolumn{2}{c}{(\ell_2 \;\; \ell_3 \;\; \ell_4 \;\; \ell_5 \;\; \ell_6 \;\; \ell_7)}
\end{array}$$

Here $l = 7$. Choose a block and form crosses between pair of lines l_i and l_j within the block with the condition that $i < j$ and no line is repeated more than once in the block. This results in exactly three crosses per block. Repeat

Table 1 Crosses from each block of BIBD

Blocks	Crosses in Each Block		
1	(ℓ_1, ℓ_2)	(ℓ_3, ℓ_4)	(ℓ_5, ℓ_6)
2	(ℓ_1, ℓ_3)	(ℓ_2, ℓ_5)	(ℓ_4, ℓ_7)
3	(ℓ_1, ℓ_7)	(ℓ_2, ℓ_4)	(ℓ_3, ℓ_6)
4	(ℓ_1, ℓ_6)	(ℓ_3, ℓ_5)	(ℓ_2, ℓ_7)
5	(ℓ_1, ℓ_4)	(ℓ_5, ℓ_7)	(ℓ_2, ℓ_6)
6	(ℓ_1, ℓ_5)	(ℓ_3, ℓ_7)	(ℓ_4, ℓ_6)
7	(ℓ_2, ℓ_3)	(ℓ_4, ℓ_5)	(ℓ_6, ℓ_7)

this process for all seven blocks such that there is no repetition of any cross that has already been chosen in the previous block(s). We have six lines and seven blocks so 21 crosses appear in the plan. The arrangement of 21 crosses in seven blocks are shown in Table 1.

We can see here that each block does not contain each of the $l(= 7)$ lines, and each cross appears exactly once across the plan. Thus, the design is a CDC plan conducted in IBD.

The $\mathcal{C}_d$ matrix of the plan is expressed as

$$\mathcal{C}_d = \left. \begin{bmatrix} 6 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 6 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 6 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 6 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 6 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 6 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 6 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 6 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 6 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 6 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 6 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 6 \end{bmatrix} \middle/ 3 \right.$$

$$= \left. \begin{bmatrix} 12 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & 12 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & 12 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & 12 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & 12 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & 12 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & 12 \end{bmatrix} \middle/ 3 \right.$$

$$\mathcal{C}_d = \frac{14}{3}\left[I_7 - \frac{1}{7}E_{77}\right].$$

Thus, for this CDC plan, $\varphi = \varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = \varphi_5 = \varphi_6 = \frac{14}{3}$ and $\varphi_7 = 0$.

The estimate of the ith line effect, $\hat{\tau}_i$, is expressed as

$$\hat{\tau}_i = \frac{1}{\varphi}Q_i = \frac{3}{14}Q_i,$$

and variance of GCA effects under CDC plan is

$$\mathrm{V}\left(\hat{\tau}_i - \hat{\tau}_j\right) = \frac{2}{\varphi}\sigma^2 = \frac{6}{14}\sigma^2.$$

On the other hand, for RCBD, variance of BLUE of GCA effects is defined as,

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{r(l-2)}\sigma^2, \quad \text{for all } i \neq j = 1, 2, \ldots, l.$$

For the given example, each cross appears once, so $r = 1$ and $l = 7$.

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{5}\sigma^2,$$

The Efficiency of CDC plan is computed as

$$\mathrm{E} = \frac{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{RCBD}}{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{CDC}} = \frac{\frac{2}{r(l-2)}\sigma^2}{\frac{2}{\varphi}\sigma^2}$$

$$= \frac{(2/5)\sigma^2}{(6/14)\sigma^2} = \frac{14}{15}.$$

The CDC plan discussed in case I is binary plan as ith line occurs once in every block for $i = 1, 2, \ldots, 7$. Next, using the same series of BIBD, we study the effect of variance of line effects and efficiency of CDC plan for non-binary case, referred to as case II. We illustrate the method of construction using the same example taken for case I.

Case II: Non-binary complete diallel cross plans

In this method, first choose one block containing k lines. Form every possible distinct pairwise crosses from k lines and place all of them in a separate block. As there are k lines in a block, this produces $\frac{k(k-1)}{2}$ distinct crosses. Even though all crosses are distinct, a line occurs multiple times within a block as it is involved in several different crosses. For this reason, the resulting CDC plan is non-binary. Repeating this procedure for all the b blocks results in a total of $\frac{bk(k-1)}{2}$ distinct crosses. As a result, we construct a non-binary CDC plan consisting of l lines distributed across b blocks. Every block includes $\frac{k(k-1)}{2}$ crosses, each line appearing $r(k-1)$ times in total and every cross occur λ times.

In this case, $\mathcal{C}_d$ matrix of the plan is expressed as

$$\mathcal{C}_d = \begin{bmatrix} (l-1)(l-2) & (l-2) & \dots & (l-2) & (l-2) \\ (l-2) & (l-1)(l-2) & \dots & (l-2) & (l-2) \\ \dots & \dots & \dots & \dots & \dots \\ (l-2) & (l-2) & \dots & (l-1)(l-2) & (l-2) \\ (l-2) & (l-2) & \dots & (l-2) & (l-1)(l-2) \end{bmatrix}$$

$$-(l-2)^2 \begin{bmatrix} (l-1 &)(l-2) & \dots & (l-2) & (l-2) \\ (l-2) & (l-1) & \dots & (l-2) & (l-2) \\ \dots & \dots & \dots & \dots & \dots \\ (l-2) & (l-2) & \dots & (l-1) & (l-2) \\ (l-2) & (l-2) & \dots & (l-2) & (l-1) \end{bmatrix} \Bigg/$$

$$\frac{(l-1)(l-2)}{2}$$

$$= (l-2)^2 \begin{bmatrix} \frac{(l-1)(l-3)}{2} & -\frac{(l-3)}{2} & \dots & -\frac{(l-3)}{2} & -\frac{(l-3)}{2} \\ -\frac{(l-3)}{2} & \frac{(l-1)(l-3)}{2} & \dots & -\frac{(l-3)}{2} & -\frac{(l-3)}{2} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{(l-3)}{2} & -\frac{(l-3)}{2} & \dots & -\frac{(l-1)(l-3)}{2} & -\frac{(l-3)}{2} \\ -\frac{(l-3)}{2} & -\frac{(l-3)}{2} & \dots & -\frac{(l-3)}{2} & \frac{(l-1)(l-3)}{2} \end{bmatrix} \Bigg/$$

$$\frac{(l-1)(l-2)}{2}$$

$$\mathcal{C}_d = \frac{l(l-2)(l-3)}{(l-1)} \left[I_l - \frac{1}{l} E_{ll} \right].$$

where, $\varphi = \frac{l(l-2)(l-3)}{(l-1)} > 0$ is the eigen value of $\mathcal{C}_d$ matrix.

4.2 Estimates and Efficiency of the Non-binary CDC Plan

We obtain the estimates and efficiency of the non-binary CDC plan.

$$\hat{\tau}_i = \frac{1}{\varphi} Q_i = \frac{1}{\frac{l(l-2)(l-3)}{(l-1)}} Q_i = \frac{(l-1)}{l(l-2)(l-3)} Q_i;$$

and under CDC plan, variance of line effects is given as

$$\mathrm{V}\left(\hat{\tau}_i - \hat{\tau}_j\right) = \frac{2}{\frac{l(l-2)(l-3)}{(l-1)}}\sigma^2 = \frac{2(l-1)}{l(l-2)(l-3)}\sigma^2.$$

And under RCBD, the variance of GCA effects can be expressed as:

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{r(l-2)}\sigma^2, \quad i \neq j = 1, 2, \ldots, l.$$

Here, $r = (l-2)$ because each cross is repeated $(l-2)$ times so

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{(l-2)^2}\sigma^2.$$

Efficiency of CDC plan compared to RCBD is defined as:

$$\mathrm{E} = \frac{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{RCBD}}{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{CDC}} = \frac{\frac{2}{(l-2)^2}\sigma^2}{\frac{2(l-1)}{l(l-2)(l-3)}\sigma^2} = \frac{l(l-3)}{(l-1)(l-2)}.$$

It can be seen that the efficiency is identical for binary as well as non-binary plans constructed from the same BIBD.

Example 4.2. Consider the BIBD from case I with $\mathrm{t} = 7, \mathrm{b} = 7, \mathrm{r} = 6, \mathrm{k} = 6$ and $\lambda = 5$. The seven blocks of this BIBD are as follows:

$$\begin{pmatrix}\ell_1 & \ell_2 & \ell_3 & \ell_4 & \ell_5 & \ell_6\end{pmatrix}, \quad \begin{pmatrix}\ell_1 & \ell_2 & \ell_3 & \ell_4 & \ell_5 & \ell_7\end{pmatrix},$$
$$\begin{pmatrix}\ell_1 & \ell_2 & \ell_3 & \ell_4 & \ell_6 & \ell_7\end{pmatrix}, \quad \begin{pmatrix}\ell_1 & \ell_2 & \ell_3 & \ell_5 & \ell_6 & \ell_7\end{pmatrix},$$
$$\begin{pmatrix}\ell_1 & \ell_2 & \ell_4 & \ell_5 & \ell_6 & \ell_7\end{pmatrix}, \quad \begin{pmatrix}\ell_1 & \ell_3 & \ell_4 & \ell_5 & \ell_6 & \ell_7\end{pmatrix},$$
$$\begin{pmatrix}\ell_2 & \ell_3 & \ell_4 & \ell_5 & \ell_6 & \ell_7\end{pmatrix}$$

Now, consider all possible distinct pairwise crosses between the lines that occur within a block. This yields $\binom{6}{2} = 15$ crosses per block. Repeat this procedure for b blocks. Thus, the number of crosses is $7 \times 15 = 105$. Also, each line appears in $r = 6$ blocks and within each block the line is paired with 5 other lines. So, each line occurs in 30 crosses. The 105 crosses arranged in seven blocks are shown in Table 2.

We can observe here that each block does not contain all the $l = 7$ lines, and each cross occur five times in the plan. Therefore, the plan is complete diallel cross plan conducted in incomplete block design. Again, each line

Table 2 One hundred five crosses in seven blocks

Block	Crosses in each block														
1	(ℓ_1,ℓ_2)	(ℓ_1,ℓ_3)	(ℓ_1,ℓ_4)	(ℓ_1,ℓ_5)	(ℓ_1,ℓ_6)	(ℓ_2,ℓ_3)	(ℓ_2,ℓ_4)	(ℓ_2,ℓ_5)	(ℓ_2,ℓ_6)	(ℓ_3,ℓ_4)	(ℓ_3,ℓ_5)	(ℓ_3,ℓ_6)	(ℓ_4,ℓ_5)	(ℓ_4,ℓ_6)	(ℓ_5,ℓ_6)
2	(ℓ_1,ℓ_2)	(ℓ_1,ℓ_3)	(ℓ_1,ℓ_4)	(ℓ_1,ℓ_5)	(ℓ_1,ℓ_7)	(ℓ_2,ℓ_3)	(ℓ_2,ℓ_4)	(ℓ_2,ℓ_5)	(ℓ_2,ℓ_7)	(ℓ_3,ℓ_4)	(ℓ_3,ℓ_5)	(ℓ_3,ℓ_7)	(ℓ_4,ℓ_5)	(ℓ_4,ℓ_7)	(ℓ_5,ℓ_7)
3	(ℓ_1,ℓ_2)	(ℓ_1,ℓ_3)	(ℓ_1,ℓ_4)	(ℓ_1,ℓ_6)	(ℓ_1,ℓ_7)	(ℓ_2,ℓ_3)	(ℓ_2,ℓ_4)	(ℓ_2,ℓ_6)	(ℓ_2,ℓ_7)	(ℓ_3,ℓ_4)	(ℓ_3,ℓ_6)	(ℓ_3,ℓ_7)	(ℓ_4,ℓ_6)	(ℓ_4,ℓ_7)	(ℓ_6,ℓ_7)
4	(ℓ_1,ℓ_2)	(ℓ_1,ℓ_3)	(ℓ_1,ℓ_5)	(ℓ_1,ℓ_6)	(ℓ_1,ℓ_7)	(ℓ_2,ℓ_3)	(ℓ_2,ℓ_5)	(ℓ_2,ℓ_6)	(ℓ_2,ℓ_7)	(ℓ_3,ℓ_5)	(ℓ_3,ℓ_6)	(ℓ_3,ℓ_7)	(ℓ_5,ℓ_6)	(ℓ_5,ℓ_7)	(ℓ_6,ℓ_7)
5	(ℓ_1,ℓ_2)	(ℓ_1,ℓ_4)	(ℓ_1,ℓ_5)	(ℓ_1,ℓ_6)	(ℓ_1,ℓ_7)	(ℓ_2,ℓ_4)	(ℓ_2,ℓ_5)	(ℓ_2,ℓ_6)	(ℓ_2,ℓ_7)	(ℓ_4,ℓ_5)	(ℓ_4,ℓ_6)	(ℓ_4,ℓ_7)	(ℓ_5,ℓ_6)	(ℓ_5,ℓ_7)	(ℓ_6,ℓ_7)
6	(ℓ_1,ℓ_3)	(ℓ_1,ℓ_4)	(ℓ_1,ℓ_5)	(ℓ_1,ℓ_6)	(ℓ_1,ℓ_7)	(ℓ_3,ℓ_4)	(ℓ_3,ℓ_5)	(ℓ_3,ℓ_6)	(ℓ_3,ℓ_7)	(ℓ_4,ℓ_5)	(ℓ_4,ℓ_6)	(ℓ_4,ℓ_7)	(ℓ_5,ℓ_6)	(ℓ_5,ℓ_7)	(ℓ_6,ℓ_7)
7	(ℓ_2,ℓ_3)	(ℓ_2,ℓ_4)	(ℓ_2,ℓ_5)	(ℓ_2,ℓ_6)	(ℓ_2,ℓ_7)	(ℓ_3,ℓ_4)	(ℓ_3,ℓ_5)	(ℓ_3,ℓ_6)	(ℓ_3,ℓ_7)	(ℓ_4,ℓ_5)	(ℓ_4,ℓ_6)	(ℓ_4,ℓ_7)	(ℓ_5,ℓ_6)	(ℓ_5,ℓ_7)	(ℓ_6,ℓ_7)

appears more than once within a block. Therefore, the resulting plan is a non-binary.

For the given example, $\mathcal{C}_d$ matrix is expressed as

$$\mathcal{C}_d = \begin{bmatrix} 30 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 30 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 30 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 30 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 30 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 30 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 30 \end{bmatrix}$$

$$- \left. \begin{bmatrix} 150 & 125 & 125 & 125 & 125 & 125 & 125 \\ 125 & 150 & 125 & 125 & 125 & 125 & 125 \\ 125 & 125 & 150 & 125 & 125 & 125 & 125 \\ 125 & 125 & 125 & 150 & 125 & 125 & 125 \\ 125 & 125 & 125 & 125 & 150 & 125 & 125 \\ 125 & 125 & 125 & 125 & 125 & 150 & 125 \\ 125 & 125 & 125 & 125 & 125 & 125 & 150 \end{bmatrix} \right/ 15$$

$$= \left. \begin{bmatrix} 300 & -50 & -50 & -50 & -50 & -50 & -50 \\ -50 & 300 & -50 & -50 & -50 & -50 & -50 \\ -50 & -50 & 300 & -50 & -50 & -50 & -50 \\ -50 & -50 & -50 & 300 & -50 & -50 & -50 \\ -50 & -50 & -50 & -50 & 300 & -50 & -50 \\ -50 & -50 & -50 & -50 & -50 & 300 & -50 \\ -50 & -50 & -50 & -50 & -50 & -50 & 300 \end{bmatrix} \right/ 15$$

$$\mathcal{C}_d = \frac{350}{15}\left[I_7 - \frac{1}{7}E_{77}\right].$$

Hence, $\varphi = \frac{350}{15}$.

The estimate of the ith line effect is

$$\hat{\tau}_i = \frac{1}{\varphi}Q_i = \frac{15}{350}Q_i,$$

and $\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)$ is obtained as

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{\varphi}\sigma^2 = \frac{30}{350}\sigma^2.$$

Under RCBD, $\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)$ is obtained as,

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{r(l-2)}\sigma^2, \quad \text{for all } i \neq j = 1, 2, \ldots, l.$$

Here, $r = 5$ and $l = 7$, so

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2}{25}\sigma^2,$$

Efficiency factor is calculated as

$$\mathrm{E} = \frac{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{RCBD}}{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)CDC} = \frac{\frac{2}{r(l-2)}\sigma^2}{\frac{2}{\varphi}\sigma^2}$$

$$= \frac{(2/25)\sigma^2}{(30/350)\sigma^2} = \frac{14}{15}.$$

From Table 3, we can notice that for the binary CDC plan, the estimate of the line effect is $\frac{3}{14}Q_i$, while for the non-binary CDC plan the estimate of the line effect is $\frac{15}{350}Q_i$ while both the CDC plan is obtained from the same BIB design. However, the efficiency factor for both the CDC plan is same as $14/15$. Now the question arises which CDC plan is to be recommended to the breeder? The simplest answer is, since the estimate of the line effect for binary CDC plan is greater than the estimate of the line effect of non-binary CDC plan, hence binary CDC plan is preferred. Again, binary CDC plan is universally optimal. Also, for binary CDC plan, total number of crosses is less than non-binary CDC plan. We can also notice that the efficiency of BIB design of this series is 0.9722 while the efficiency of CDC plan obtained from this series of BIB design is 0.9333. This shows that efficiency factor of both the design is more or less same and is very high. Finally, we can say that efficiency factor of BIB design obtained from this series is always very high and near to one.

Table 3 Efficiency factor of BIB design and CDC plans with $l = 7$ lines

t	b	r	k	λ	Efficiency of BIBD	Type of CDC plan	Total number of cross	Number of times a cross is repeated	Estimates of the line effect	$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{RCBD}$	$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{CDC}$	Efficiency of CDC
7	7	6	6	5	35/36	Binary	21	1	$\frac{3}{14}Q_i$	$(2/5)\sigma^2$	$(6/14)\sigma^2$	$14/15 = .9333$
7	7	6	6	5	35/36	Non-binary	105	5	$\frac{15}{350}Q_i$	$(2/25)\sigma^2$	$(30/350)\sigma^2$	$14/15 = .9333$

5 Optimality of CDC Plan

In this section, we outline the universal optimality criterion for CDC experimental designs. Let $\mathcal{D}(t, b, k)$ denotes a class of block designs $\{\mathrm{d}\}$ for diallel crosses with t treatments and b blocks each of size k. Assume that the experiment involves l parental lines, leading to a total of $\frac{l(l-1)}{2}$ crosses. The aim is to construct an IBD in which $\frac{l(l-1)}{2}$ crosses are distributed across b blocks, each containing k crosses. Let $n = bk$ be the total number of experimental units in the design d. The optimality criterion for CDC plan is based on minimizing the average variance of the BLUE of all pairwise comparisons between GCA effects. In particular, a design that is universally optimal is also A-optimal. Using the above facts and Kiefer (1958, 1975), we have the following lemma:

Lemma 1. Consider a block design $\mathrm{d} \in \mathcal{D}(t, b, k)$ for a diallel crosses. Suppose that the $\mathcal{C}_d$ matrix of d is completely symmetric, that is, all the diagonal entries as well as off-diagonal entries of matrix C_d are equal and

$$\mathrm{Tr}(\mathcal{C}_d) \leq k^{-1}b\{2k(k-1-2x) + tx(x+1)\}$$

where, $x = [\frac{2k}{t}]$, $[\cdot]$ is the greatest integer function and $\mathrm{Tr}(A)$ stands for trace of a square matrix A.

If this condition is satisfied, then the plan is universally optimal in $\mathcal{D}(t, b, k)$. This lemma is due to Dey (2002).

Note that when $2k < t$, we have $x = 0$ and in this case,

$$\mathrm{Tr}(\mathcal{C}_d) \leq 2b(k-1)$$

Now we can check the universal optimality of complete diallel crosses plan constructed in Example 4.1. For this example, $\mathrm{Tr}(\mathcal{C}_d) = 28$ and $k^{-1}b\{2k(k-1-2x) + tx(x+1)\} = 28$. Thus, the condition of Lemma 1 is satisfied. Hence, the constructed complete diallel crosses plan is universally optimal.

6 Robustness of Binary Complete Diallel Cross Plan

We also carried out the robustness of residual CDC plan after losing one block. Consider the binary CDC plan discussed in Case 1 in Section 4. Let us assume that we lost the last block of the binary CDC plan. In this case, the last block does not contain line ℓ_1. The residual CDC plan is with l lines, each line is replicated $(l-2)$ times except line ℓ_1 which is replicated $(l-1)$ times.

Table 4 Distinct crosses arranged in $(l-1)$ blocks for residual CDC plan

Blocks	Crosses in Each Block					
1	(ℓ_1, ℓ_2)	(ℓ_3, ℓ_4)	(ℓ_5, ℓ_6)	$\dots$	(ℓ_{l-4}, ℓ_{l-3})	(ℓ_{l-2}, ℓ_{l-1})
2	(ℓ_1, ℓ_3)	(ℓ_2, ℓ_5)	(ℓ_4, ℓ_7)	$\dots$	(ℓ_{l-5}, ℓ_{l-2})	(ℓ_{l-3}, ℓ_l)
3	(ℓ_1, ℓ_7)	(ℓ_2, ℓ_4)	(ℓ_3, ℓ_6)	$\dots$	(ℓ_{l-5}, ℓ_{l-3})	(ℓ_{l-4}, ℓ_{l-1})
4	(ℓ_1, ℓ_6)	(ℓ_3, ℓ_5)	(ℓ_2, ℓ_7)	$\dots$	(ℓ_{l-4}, ℓ_{l-2})	(ℓ_{l-5}, ℓ_l)
5	(ℓ_1, ℓ_4)	(ℓ_5, ℓ_7)	(ℓ_2, ℓ_6)	$\dots$	(ℓ_{l-2}, ℓ_l)	(ℓ_{l-5}, ℓ_{l-1})
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$l-2$	(ℓ_1, ℓ_5)	(ℓ_3, ℓ_7)	(ℓ_4, ℓ_6)	$\dots$	(ℓ_{l-4}, ℓ_l)	(ℓ_{l-3}, ℓ_{l-1})
$l-1$	(ℓ_2, ℓ_3)	(ℓ_4, ℓ_5)	(ℓ_6, ℓ_7)	$\dots$	(ℓ_{l-3}, ℓ_{l-2})	(ℓ_{l-1}, ℓ_l)

The $\frac{l(l-1)}{2} - \frac{l-1}{2} = \frac{(l-1)^2}{2}$ distinct crosses are arranged in $(l-1)$ blocks each of size $\frac{(l-1)}{2}$ crosses as shown in Table 4.

The C_d matrix for the residual CDC plan is expressed as

$$C_d = \begin{bmatrix} (l-1) & 1 & 1 & \dots & 1 & 1 \\ 1 & (l-2) & 0 & \dots & 1 & 1 \\ 1 & 0 & (l-2) & \dots & 1 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & (l-2) & 0 \\ 1 & 1 & 1 & \dots & 0 & (l-2) \end{bmatrix}$$

$$- \begin{bmatrix} (l-1) & (l-2) & (l-2) & \dots & (l-2) & (l-2) \\ (l-2) & (l-2) & (l-3) & \dots & (l-3) & (l-3) \\ (l-2) & (l-3) & (l-2) & \dots & (l-3) & (l-3) \\ \dots & \dots & \dots & \dots & \dots & \dots \\ (l-2) & (l-3) & (l-3) & \dots & (l-2) & (l-3) \\ (l-2) & (l-3) & (l-3) & \dots & (l-3) & (l-2) \end{bmatrix} \Bigg/ \frac{(l-1)}{2}$$

$$= \begin{bmatrix} \frac{(l-1)(l-3)}{2} & -\frac{(l-3)}{2} & -\frac{(l-3)}{2} & \dots & -\frac{(l-3)}{2} & -\frac{(l-3)}{2} \\ -\frac{(l-3)}{2} & \frac{(l-2)(l-3)}{2} & -\frac{(l-3)}{1} & \dots & -\frac{(l-5)}{2} & -\frac{(l-5)}{2} \\ -\frac{(l-3)}{2} & -\frac{(l-3)}{1} & \frac{(l-2)(l-3)}{2} & \dots & -\frac{(l-5)}{2} & -\frac{(l-5)}{2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ -\frac{(l-3)}{2} & -\frac{(l-5)}{2} & -\frac{(l-5)}{2} & \dots & \frac{(l-2)(l-3)}{2} & -\frac{(l-3)}{1} \\ -\frac{(l-3)}{2} & -\frac{(l-5)}{2} & -\frac{(l-5)}{2} & \dots & -\frac{(l-3)}{1} & \frac{(l-2)(l-3)}{2} \end{bmatrix} \Bigg/ \frac{(l-1)}{2}$$

The non-zero eigen values of C_d – matrix are $\varphi_i = \frac{l(l-3)}{l-1}$ with multiplicity $\frac{(l+1)}{2}$ and $\varphi_j = \frac{l^2-5l+2}{l-1}$ with multiplicity $\frac{(l-3)}{2}$. That is, $\varphi_1 = \varphi_2 = \dots = \varphi_{\frac{(l+1)}{2}} = \frac{l(l-3)}{l-1}$ and $\varphi_{\frac{(l+1)}{2}+1} = \dots = \varphi_{l-1} = \frac{l^2-5l+2}{l-1}$.

The variance of the line effects under residual CDC plan is given as

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \left(\frac{l(l-3)}{l-1} \times \frac{(l+1)}{2} + \frac{l^2-5l+2}{l-1} \times \frac{(l-3)}{2} \right) \Big/ \left(\frac{(l+1)}{2} + \frac{(l-3)}{2} \right) \sigma^2$$

$$= \left(\frac{l+1}{l(l-3)} + \frac{(l-3)}{l^2-5l+2} \right) \sigma^2$$

$$= \frac{2}{l(l-3)\,(l^2-5l+2)} \left[l^2(l-5) + (3l+1) \right] \sigma^2.$$

The variance of any elementary contrast among GCA effects under binary CDC plan is (computed as per case I) given by,

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{2(l-1)}{l(l-3)} \sigma^2, \quad \text{for all } i \neq j = 1, 2, \ldots, l.$$

The Efficiency factor of the residual CDC plan compared to binary CDC plan is obtained as

$$\mathrm{E} = \frac{\mathrm{V}(\hat{g}_i - \hat{g}_j)_{CDC}}{\mathrm{V}(\hat{g}_i - \hat{g}_j) resiual_{CDC}} = \frac{\frac{2(l-1)}{l(l-3)} \sigma^2}{\frac{2}{l(l-3)(l^2-5l+2)} [l^2(l-5) + (3l+1)] \sigma^2}$$

After simplification, we have

$$\mathrm{E} = \frac{(l-1)(l^2-5l+2)}{[l^2(l-5) + (3l+1)]}$$

Example 4.3. Consider the BIBD constructed in Example 4.1 and delete the last block. Then the plan becomes

Blocks	Crosses in Each Block		
1	(ℓ_1, ℓ_2)	(ℓ_3, ℓ_4)	(ℓ_5, ℓ_6)
2	(ℓ_1, ℓ_3)	(ℓ_2, ℓ_5)	(ℓ_4, ℓ_7)
3	(ℓ_1, ℓ_7)	(ℓ_2, ℓ_4)	(ℓ_3, ℓ_6)
4	(ℓ_1, ℓ_6)	(ℓ_3, ℓ_5)	(ℓ_2, ℓ_7)
5	(ℓ_1, ℓ_4)	(ℓ_5, ℓ_7)	(ℓ_2, ℓ_6)
6	(ℓ_1, ℓ_5)	(ℓ_3, ℓ_7)	(ℓ_4, ℓ_6)

The residual CDC plan is with 7 lines, each line is replicated 5 times except line ℓ_1 which is replicated 6 times. Eighteen distinct crosses are arranged in six blocks each of three crosses. We compute the variance of the line effects of the residual CDC plan using the C_d matrix.

$$C_d = \begin{bmatrix} 6 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 5 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 5 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 5 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 5 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 5 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 4 & 4 & 4 & 4 & 4 \\ 5 & 4 & 5 & 4 & 4 & 4 & 4 \\ 5 & 4 & 4 & 5 & 4 & 4 & 4 \\ 5 & 4 & 4 & 4 & 5 & 4 & 4 \\ 5 & 4 & 4 & 4 & 4 & 5 & 4 \\ 5 & 4 & 4 & 4 & 4 & 4 & 5 \end{bmatrix} \Bigg/ 3$$

$$= \begin{bmatrix} 12 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & 10 & -4 & -1 & -1 & -1 & -1 \\ -2 & -4 & 10 & -1 & -1 & -1 & -1 \\ -2 & -1 & -1 & 10 & -4 & -1 & -1 \\ -2 & -1 & -1 & -4 & 10 & -1 & -1 \\ -2 & -1 & -1 & -1 & -1 & 10 & -4 \\ -2 & -1 & -1 & -1 & -1 & -4 & 10 \end{bmatrix} \Bigg/ 3$$

The non-zero eigen values of C_d matrix are $\varphi = \frac{14}{3}$ with multiplicity 4 and $\varphi = \frac{8}{3}$ with multiplicity 2. That is, $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = \frac{14}{3}$, $\varphi_5 = \varphi_6 = \frac{8}{3}$, and $\varphi_7 = 0$.

The estimates of $\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)$ under residual CDC plan is given as

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \left(\frac{6}{14} \times 4 + \frac{6}{8} \times 2\right) \Big/ (4+2)\sigma^2 = \frac{45}{14 \times 6}\sigma^2 = \frac{15}{28}\sigma^2.$$

The variance $\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)$ under binary CDC plan (computed as per Example 4.1) is given by,

$$\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j) = \frac{6}{14}\sigma^2, \quad \text{for all } i \neq j = 1, 2, \ldots, l.$$

For the residual CDC plan, the efficiency factor is given as

$$\mathrm{E} = \frac{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)_{CDC}}{\mathrm{V}(\hat{\tau}_i - \hat{\tau}_j)resiual_{CDC}} = \frac{\frac{6}{14}\sigma^2}{\frac{15}{28}\sigma^2} = \frac{4}{5} = 0.8.$$

That is, efficiency factor of residual CDC plan compared to binary CDC plan is 80 percent. Hence, we lost only twenty percent information on residual

CDC plan after losing one complete block. In other word, the residual CDC plan resulting from the loss of a single block is highly robust with efficiency factor 0.8.

7 Conclusion

Although binary and non-binary complete diallel cross plans derived from same series of BIB designs with identical number of lines exhibit the same efficiency factor, notable differences exist between them. In particular, total number of crosses for non-binary CDC plan is more compared to binary CDC plan. Despite this, the estimates of GCA effects for the lines is larger for binary CDC plan compared to those for non-binary CDC plan. Furthermore, the binary CDC plan obtained from $t = b = l, r = k = l - 1$ and $\lambda = l - 2$ series of balanced incomplete block design is universally optimal. These designs exhibit strong robustness against the loss of one block, making them more practical for breeding experiments.

Conflict of Interest

The authors have no conflicts of interest to declare.

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