Generalized Memory-Type Estimators for Population Mean Under SRSWOR

Prayas Sharma[1], Rajesh Singh[2] and Yashveer Singh[2,*]

[1]*Department of Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow, 226025, India*
[2]*Department of Statistics, Banaras Hindu University, Varanasi, 221005, India*
E-mail: prayassharma02@gmail.com; rsinghstat@gmail.com; yashveerbhu@gmail.com
[*]*Corresponding Author*

Received 12 June 2026; Accepted 02 August 2026

Abstract

In classical survey sampling, the population mean is typically estimated using fixed observed values, often incorporating auxiliary information to improve precision. The primary objective of this study is to develop a family of memory-type estimators based on the Exponentially Weighted Moving Average (EWMA) technique for estimating the population mean. The proposed methodology exploits historical sample information through EWMA statistic, thereby enhancing the efficiency of estimation. A memory-based estimator incorporating auxiliary information is formulated under the framework of Simple Random Sampling Without Replacement (SRSWOR). The performance of the proposed family of estimators is evaluated through an extensive empirical investigation. The results demonstrate that the use of EWMA statistic significantly improves the precision of population mean estimation compared to conventional estimators. Furthermore, the

Journal of Reliability and Statistical Studies, Vol. 19, Issue 2 (2026), 587–614.
doi: 10.13052/jrss0974-8024.19214

efficiency of the proposed estimators can be controlled and optimized through appropriate selection of the smoothing parameter, δ. The practical applicability of the proposed methodology is illustrated using a real-world dataset.

Keywords: Exponentially weighted moving average statistic (EWMA), mean estimation, mean squared error, study variable, auxiliary variable.

1 Introduction

In survey sampling, the primary objective is to estimate population parameters, such as the mean, median, and variance, with the highest possible precision. One of the most effective approaches for improving the accuracy of these estimates is the incorporation of auxiliary information (AUI), which is obtained from one or more auxiliary variables that are correlated with the study variable. By exploiting the relationship between the study and auxiliary variables, auxiliary information provides additional insight into the population structure and enables the construction of more efficient estimators. The use of auxiliary information has been extensively studied in the survey sampling literature and forms the basis of several classical estimation techniques, including the ratio, product, and regression estimators. These estimators utilize the known characteristics of the auxiliary variable(s) to reduce sampling variance, thereby producing more precise estimates than estimators that rely solely on the study variable.

The ratio estimator, introduced by [1], leverages the known population mean of an auxiliary variable X, assuming a strong positive correlation between X and the study variable Y. The estimator is given by:

$$\widehat{\overline{Y}}_r = \frac{\bar{y}}{\bar{x}} \cdot \overline{X} \tag{1}$$

where $\overline{y}$ and $\bar{x}$ are the sample means of the study and auxiliary variables, respectively, and $\bar{X}$ is the known population mean of the auxiliary variable. This method is particularly effective when the regression line between X and Y is linear and passes through the origin. Conversely, the product estimator, proposed by [2], is appropriate when the study and auxiliary variables are negatively correlated:

$$\widehat{\overline{Y}}_p = \frac{\bar{y}}{\bar{X}} \cdot \bar{x} \tag{2}$$

Both estimators have respective mean square error (MSE) expressions that quantify their performance under varying correlation structures:

$$MSE\left(\widehat{\overline{Y}}_r\right) = f\overline{Y}^2\left[C_y^2 + C_x^2 - 2\rho C_x C_y\right]. \quad (3)$$

$$MSE\left(\widehat{\overline{Y}}_p\right) = f\bar{Y}^2\left[C_y^2 + C_x^2 + 2\rho C_x C_y\right]. \quad (4)$$

Here, C_y and C_x denote the coefficients of variation of the study and auxiliary variables, ρ_{xy} is the correlation coefficient, and $f = \frac{1}{n} - \frac{1}{N}$ for a population of size N and sample size n.

Several researchers have proposed modifications and improvements to these estimators to enhance their applicability under different sampling designs. For instance, [3] suggested transformation-based ratio estimators to adjust for cases where linearity does not hold strictly, and [4] introduced hybrid ratio and product estimators by combining current and past sample information. While these classical methods provide significant improvements, they often fail to address the dynamic nature of data collected over time. In repeated surveys, longitudinal studies, and industrial quality monitoring, observations are not static and may be subject to gradual shifts or time-dependent changes. In such scenarios, EWMA estimators offer a more flexible and responsive approach.

Several researchers have tried to improve mean estimation efficiency through different techniques such as [5] introduced a logarithmic ratio-type estimator for estimating the finite population mean under the neutrosophic framework, demonstrating improved efficiency in the presence of uncertainty, [6] developed an almost unbiased estimator for the population mean using neutrosophic information, thereby reducing estimation bias while maintaining high precision, [7] proposed an estimator utilizing two neutrosophic auxiliary variables under imprecise information, showing enhanced estimation accuracy compared with existing methods, [8] introduced enhanced memory-type estimators incorporating dual auxiliary variables, illustrating the benefits of memory-based estimation strategies for improving estimator efficiency. These studies collectively demonstrate the growing interest in developing efficient estimators using auxiliary information and provide the motivation for the proposed EWMA-based estimator presented in this paper, [9] introduced optimized estimators for the population mean under stratified random sampling by incorporating linear cost structures. Their study derived optimum allocation strategies and demonstrated that the proposed estimators achieve higher efficiency than conventional estimators under varying survey cost constraints, [10] and [23] proposed advanced memory-type exponential estimators for population mean estimation by integrating the Exponentially Weighted Moving Average (EWMA) framework with auxiliary information and auxiliary attributes. They demonstrated, through theoretical derivations and simulation studies, that the proposed estimators provide higher efficiency and lower mean squared error than existing exponential estimators, [11, 21, 22] and [12] developed a class of exponential ratio estimators using two auxiliary variables and showed that the proposed estimators achieve improved efficiency compared with existing population mean estimators.

EWMA techniques originated in statistical process control and are widely recognized for their ability to detect small, sustained changes in the process mean. Unlike traditional charts, which give equal weight to all past data, EWMA assigns exponentially decreasing weights to older observations. The EWMA statistic at time t is defined as:

$$W_t = \delta \bar{y}_t + (1 - \delta) W_{t-1}. \tag{5}$$

$$Z_t = \delta \bar{x}_t + (1 - \delta) Z_{t-1}. \tag{6}$$

where $0 < \delta \leq 1$ is the smoothing constant, $\bar{y}_t$ is the sample mean at time t, and W_{t-1} is the EWMA from the previous time step. A smaller value of δ allows the estimator to incorporate a longer memory of past data, while a larger δ makes the estimator more responsive to recent changes.

The integration of EWMA methods into survey sampling has emerged as a robust approach for improving estimation accuracy, particularly in scenarios involving time-ordered or sequential data collection. EWMA-based estimators incorporate both current and past sample information by assigning more weight to recent data points, making them particularly effective in detecting gradual changes in population characteristics over time. This property enables EWMA-based estimators to maintain lower variance and bias compared to static classical methods, especially in the presence of auto correlation or evolving trends. Their application is especially valuable in domains such as monthly household surveys, economic forecasting, and epidemiological monitoring, where data are collected periodically and population dynamics shift over time.

The present study proposes a family of EWMA-type estimators that utilize auxiliary information under various correlation structures using SRSWOR. By incorporating both temporal sample dynamics and auxiliary correlations, these estimators provide improved efficiency compared to their classical counterparts. Theoretical properties such as bias, mean square error, and relative efficiency are derived, and the performance is further validated through simulation studies and real-data applications. The findings establish EWMA-based techniques as a superior alternative in time-sensitive survey settings.

The primary contribution of this paper is the development of a generalized EWMA-based family of memory-type estimators for population mean estimation under SRSWOR. The proposed family effectively incorporates auxiliary information through a functional form, allowing several estimators to be obtained as special cases under this class of estimator. Theoretical properties of the proposed estimators are derived, and their performance is validated through empirical and simulation studies, demonstrating superior performance.

2 Exponentially Weighted Moving Average (EWMA) Statistic

To develop the proposed family of estimators,the EWMA statistics employed in the subsequent theoretical derivations are introduced in the following section. For $t > 0$, the EWMA statistic used for estimating the population mean is formulated based on the sample means of the study variable and the associated auxiliary variables.

$$W_t = \delta\bar{y}_t + (1-\delta)W_{t-1}. \tag{7}$$

$$Z_t = \delta\bar{x}_t + (1-\delta)Z_{t-1}. \tag{8}$$

$$E(W_t) = \mu_y \tag{9}$$

$$\begin{aligned} Var(W_t) &= Var\left[\delta\bar{y}_t + (1-\delta)(\delta\bar{y}_{t-1} + (1-\delta)V_{t-2})\right] \\ &= Var\left[\delta\bar{y}_t + (1-\delta)\delta\bar{y}_{t-1} + (1-\delta)^2\delta\bar{y}_{t-2} + \cdots\right] \\ &= Var(\bar{y}_t)\delta^2\left[1 + (1-\delta)^2 + (1-\delta)^4 + \cdots\right] \\ &= Var(\bar{y}_t)\delta^2\sum_{n=0}^{\infty}(1-\delta)^{2n}. \end{aligned} \tag{10}$$

Hence, the limiting variance of EWMA statistic W_t is given by

$$Var(W_t) = \frac{\delta^2 Var(\bar{y}_t)}{2\delta - \delta^2} = \Delta Var(\bar{y}_t). \tag{11}$$

Similarly,

$$E(Z_t) = \mu_x \tag{12}$$

$$Var(Z_t) = \frac{\delta^2 Var(\bar{x}_t)}{2\delta - \delta^2} = \Delta Var(\bar{x}_t). \tag{13}$$

In order to derive the Mean Squared Error (MSE) of the proposed memory-type estimators, the following notations are considered throughout the analysis.

$$W_t = \bar{Y}(1 + e_{0t}), \quad Z_t = \bar{X}(1 + e_{1t}) \tag{14}$$

such that

$$E(e_{0t}) = E(e_{1t}) = 0 \tag{15}$$

$$E(e_{0t}^2) = \Delta f C_y^2 \tag{16}$$

$$E(e_{1t}^2) = \Delta f C_x^2 \tag{17}$$

$$E(e_{0t}e_{1t}) = \Delta f \rho_{xy} C_y C_x \tag{18}$$

where, $\Delta = \frac{\delta}{2-\delta}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$ and $C_x^2 = \frac{S_x^2}{\bar{X}^2}$ are the coefficient of variation of Y and X. $S_y^2 = \sum_{i=1}^{N} \frac{(Y_i - \overline{Y})^2}{N-1}$, $S_x^2 = \frac{\sum_{i=1}^{N}(X_i - \overline{X})^2}{N-1}$ are the population variances of study variable and auxiliary variable respectively. $\rho_{xy} = \frac{S_{xy}}{\sqrt{S_x{}^2 S_y{}^2}}$ is the correlation coefficient between X and Y.

3 Review of Estimators

Based on the EWMA framework introduced in the previous section, the existing memory-type estimators reported in the literature are briefly reviewed to establish the foundation for the proposed methodology.

(a) [13] suggested memory-type ratio and product estimators as follows:

$$T_0 = \bar{y} \tag{19}$$

$$MSE\,(T_0) = f\, C_y^2\, \bar{Y}^2. \tag{20}$$

$$T_1 = \frac{Z_t}{W_t} \cdot \bar{X} \tag{21}$$

$$T_2 = \frac{Z_t}{\bar{X}} \cdot W_t \tag{22}$$

$$MSE\left(T_1\right) = \Delta f \overline{Y}^2 ({C_y}^2 + {C_x}^2 - 2\rho C_x C_y). \tag{23}$$

$$MSE\left(T_2\right) = \Delta f \overline{Y}^2 \left({C_y}^2 + {C_x}^2 + 2\rho C_x C_y\right). \tag{24}$$

(b) Regression estimator was suggested by [14] and motivated by this [15] suggested the below given estimator

$$T_3 = Z_t + b(\bar{X} - W_t) \tag{25}$$

The approximate MSE is given by:

$$MSE(T_3) = \Delta f \bar{Y}^2 {C_y}^2 (1 - \rho^2). \tag{26}$$

(c) The exponential ratio-type estimator was originally proposed by [16]. Motivated by their work, [15] subsequently suggested the following estimator:

$$T_4 = Z_t exp\left(\frac{\overline{X} - W_t}{\bar{X} + W_t}\right) \tag{27}$$

The approximate MSE will be:

$$MSE(T_4) = \Delta f \bar{Y}^2 \left({C_y}^2 + {C_x}^2\left(\frac{1}{4} - \frac{\rho C_y}{C_x}\right)\right). \tag{28}$$

(d) Suggested by [17] given below

$$T_5 = \frac{E_{yt}}{E_{xt}} \cdot \bar{X} \tag{29}$$

$$E_{yt} = \delta \bar{y}_{regt} + (1 - \delta) E_{y(t-1)} \tag{30}$$

$$E_{xt} = \delta \bar{x}_t + (1 - \delta) E_{x(t-1)} \tag{31}$$

$$MSE(T_5) = \Delta f \bar{Y}^2 ({C_y}^2 \left(1 - \rho^2\right) + {C_x}^2). \tag{32}$$

(e) Memory type estimator for estimating population mean using time-scaled survey as suggested by [18]

$$T_6 = \left[w_1 W_t \left(\frac{\mu_x}{Z_t}\right)^{\alpha} \exp\left(\frac{\eta(\mu_x - Z_t)}{\eta(\mu_x + Z_t) + 2\theta}\right)\right] + w_2 Z_t + (1 - w_1 - w_2)\mu_x \tag{33}$$

where $\theta = \frac{1}{n} - \frac{1}{N}$ and α are real numbers or functions of the known parameters of auxiliary variable

$$\mathrm{MSE}(T_6) = (1 - 2w_1)b^2 + w_1^2 A + w_2^2 B + 2w_1 w_2 C. \tag{34}$$

$$\text{Let } A = b^2 + \mu_y^2\theta\left(\frac{\delta}{2-\delta}\right)\left(C_y^2 + a^2C_x^2 - 2a\rho C_y C_x\right) \tag{34a}$$

$$B = \mu_x^2\theta C_x^2\left(\frac{\delta}{2-\delta}\right) \tag{34b}$$

$$C = \mu_x\mu_y\theta\left(\frac{\delta}{2-\delta}\right)\left(\rho C_y C_x - aC_x^2\right) \tag{34c}$$

where w_1 and w_2 are, respectively.

$$w_1 = \frac{Bb^2}{AB - C^2} \tag{34d}$$

$$w_2 = \frac{-Cb^2}{AB - C^2} \tag{34e}$$

where, a and k are

$$a = k + \alpha \tag{34f}$$

$$k = \frac{\eta\mu_x}{2(\eta\mu_x + \theta)} \tag{34g}$$

where, η is a real number

(f) Estimation of population mean with the use of memory type estimator as suggested by [15]

$$T_7 = \left\{\vartheta_1 W_t + \vartheta_2\left(\frac{W_t}{Z_t}\right)\bar{X}\right\}\exp\left[\frac{\alpha(\bar{X} - Z_t)}{\alpha(\bar{X} + Z_t) + 2\beta}\right] \tag{35}$$

where, α and β are real constants

$$\text{MSE}(T_7) = A_m\vartheta_1^2 + B_m\vartheta_2^2 + 2C_m\vartheta_1\vartheta_2 + 2D_m\vartheta_1 + 2E_m\vartheta_2 + F_m. \tag{36}$$

where

$$A_m = \bar{Y}^2\left[1 + f_1\left(\frac{\delta}{2-\delta}\right)\left\{C_y^2 + C_x^2 - 2\rho C_y C_x\right\}\right] \tag{36a}$$

$$B_m = \bar{Y}^2\Bigg[1 + f_1\left(\frac{\delta}{2-\delta}\right)\{C_y^2 + (3 + 4\gamma + 4\gamma^2) \times C_x^2 - 4(1+\gamma)\rho C_y C_x\}\Bigg] \tag{36b}$$

$$C_m = \bar{Y}^2 \Bigg[1 + f_1 \left(\frac{\delta}{2-\delta} \right) \{ C_y^2 + (1 + 2\gamma + 4\gamma^2) C_x^2 - 2(1+2\gamma)\rho C_y C_x \} \Bigg] \tag{36c}$$

$$D_m = -\bar{Y}^2 \left[1 + f_1 \left(\frac{\delta}{2-\delta} \right) \left\{ \tfrac{3}{2}\gamma^2 C_x^2 - \rho C_y C_x \right\} \right] \tag{36d}$$

$$E_m = -\bar{Y}^2 \Bigg[1 + f_1 \left(\frac{\delta}{2-\delta} \right) \times \left\{ \left(1 + \gamma + \tfrac{3}{2}\gamma^2\right) C_x^2 - (1-\gamma)\rho C_y C_x \right\} \Bigg] \tag{36e}$$

$$F_m = \bar{Y}^2 \tag{36f}$$

where, ϑ_1 and ϑ_2, we have

$$\vartheta_1 = \frac{B_m D_m - C_m E_m}{C_m^2 - A_m B_m} \tag{36g}$$

$$\vartheta_2 = \frac{A_m E_m - C_m D_m}{C_m^2 - A_m B_m} \tag{36h}$$

$$\gamma = \frac{\alpha \bar{X}}{2(\beta + \alpha \bar{X})} \tag{36i}$$

4 Proposed Estimator

In this section, we propose a generalized family of memory-type estimators designed for the estimation of the population mean by utilizing information from a single auxiliary variable. The development of this family is carried out under the framework of Simple Random Sampling Without Replacement (SRSWOR).

The proposed formulation aims to extend existing estimators by introducing a flexible structure that can efficiently incorporate auxiliary information through memory-type components. This generalization not only encompasses several well-known estimators as special cases but also provides a broader platform for constructing new estimators with improved efficiency and reduced mean squared error under different population settings:

$$T_g = \delta \left[W_t + k_1 W_t \cdot \left(\frac{\bar{Z}}{\alpha Z_t + (1-\alpha)\bar{Z}} \right) + k_2 \left(\bar{Z} - Z_t \right) \right] \cdot \left(\frac{\bar{Z}}{\alpha Z_t + (1-\alpha)\bar{Z}} \right)^g \tag{37}$$

where $\bar{Z} = a\bar{X} + b$, here, k_1, k_2 and δ are appropriate constants to be identified such that the MSE of t_g is minimum, the quantities a, b, and δ represent either fixed real constants or functions of known parameters related to the auxiliary variable and $k_1 + k_2 \neq 1$, expressing in terms of e_{0t}, e_{1t} we get,

$$T_g = \delta \left[\overline{Y}\left(1 + e_{0t}\right) + k_1 \overline{Y}(1 + e_{0t}) \cdot \left(\frac{a\bar{X} + b}{\alpha \left(a\overline{X}\left(1 + e_{1t}\right) + b \right)} \right) + k_2 \left(\frac{\alpha \left(a\overline{X}\left(1 + e_{1t}\right) + b \right) + (1-\alpha)\left(a\overline{X} + b \right)}{\alpha \left(a\overline{X} + b \right)} \right)^g \right] \tag{38}$$

$$T_g = \delta \left[\bar{Y}\left(1 + e_{0t}\right) + k_1 \bar{Y}(1 + e_{0t}) \left(1 - \alpha\theta e_{1t} + \alpha^2\theta^2 e_{1t}^2 \right) + k_2 \left(-\alpha \bar{X} e_{1t} \right) \left[1 - g\alpha\theta e_{1t} + \frac{g(g+1)}{2}\theta^2\alpha^2 e_{1t}^2 \right] \right]. \tag{39}$$

where $\theta = \frac{a\overline{X}}{a\overline{X}+b}$, expanding using Taylor's expansion up to first order approximation, we have

$$\begin{aligned} T_g - \overline{Y} = {} & \delta\overline{Y} \left(1 - \alpha\theta g e_{1t} + \frac{g(g+1)}{2}\alpha^2\theta^2 e_{1t}^2 + e_{0t} - \alpha\theta g e_{1t} e_{0t} \right) \\ & + \delta\overline{Y} k_1 \Big(1 - \alpha\theta g e_{1t} + \frac{g(g+1)}{2}\alpha^2\theta^2 e_{1t}^2 - \alpha\theta g e_{1t} + \alpha^2\theta^2 g e_{1t}^2 \\ & + \alpha^2\theta^2 e_{1t}^2 + e_{0t} - \alpha\theta g e_{1t} e_{0t} \Big) - \delta k_2 a\overline{X} \left(e_{1t} - \alpha\theta g e_{1t}^2 \right) - \overline{Y}. \end{aligned} \tag{40}$$

squaring, and taking expectation on both sides we get the MSE of the estimator T_g is given by

$$MSE(T_g) = \delta^2 \left[{k_1}^2 A - 2k_1 k_2 B + 2k_1 C + {k_2}^2 D - 2k_2 E + F \right] - 2\delta \left[k_1 G + k_2 H + I \right] + \overline{Y}^2 \tag{41}$$

where,

$$A = \overline{Y}^2\Big[1+\alpha^2\theta^2(g+1)^2\Delta f{C_x}^2+\Delta f{C_y}^2-2\alpha\theta(g+1)$$
$$\times\Big(-\alpha\theta\Delta f{C_x}^2+\Delta f\rho C_yC_x\Big)$$
$$+g(g+1)\alpha^2\theta^2\Delta f{C_x}^2-2\alpha\theta(g+1)\Delta f\rho C_yC_x\Big] \quad (42)$$

$$B = a\,\overline{Y}\,\overline{X}\Big[-\alpha\theta\Delta f{C_x}^2-\alpha\theta(g+1)\Delta f{C_x}^2+\Delta f\rho C_yC_x\Big] \quad (43)$$

$$C = \Big[\overline{Y}^2+\overline{Y}^2\big(-\alpha\theta\Delta f\rho C_yC_x\big)$$
$$-\overline{Y}^2\alpha\theta(g+1)\Big(-\alpha\theta\Delta f{C_x}^2+\Delta f\rho C_yC_x\Big)$$
$$-\overline{Y}^2\alpha\theta(g+1)\Big(\Delta f\rho C_yC_x-\alpha\theta g\Delta f{C_x}^2\Big)$$
$$+2\overline{Y}^2g(g+1)\alpha^2\theta^2\Delta f{C_x}^2$$
$$+\overline{Y}^2\Big(\Delta f{C_y}^2-\alpha\theta g\Delta f\rho C_yC_x\Big)\Big] \quad (44)$$

$$D = \Big[a^2\overline{X}^2\Delta f{C_x}^2\Big] \quad (45)$$

$$E = \Big[a\overline{Y}\,\overline{X}\left(\Delta f\rho C_yC_x-\alpha\theta g\Delta f{C_x}^2\right)-a\overline{Y}\,\overline{X}\alpha\theta g\Delta f{C_x}^2\Big] \quad (46)$$

$$F = \Big[\overline{Y}^2+\overline{Y}^2\left(\Delta f{C_y}^2+\alpha^2\theta^2\Delta f{C_x}^2-2\alpha\theta g\Delta f\rho C_yC_x\right)$$
$$+2\overline{Y}^2\left(-\alpha\theta g\Delta f\rho C_yC_x\right)+\overline{Y}^2g(g+1)\alpha^2\theta^2\Delta f{C_x}^2\Big] \quad (47)$$

$$G = \Big[\overline{Y}^2-\overline{Y}^2\alpha\theta g(g+1)\left(-\alpha\theta\Delta f{C_x}^2+\Delta f\rho C_yC_x\right)$$
$$+\overline{Y}^2\frac{g(g+1)}{2}\alpha^2\theta^2\Delta f{C_x}^2\Big] \quad (48)$$

$$H = \left[a\overline{Y}\,\overline{X}\alpha\theta g\Delta f{C_x}^2\right] \quad (49)$$

$$I = \Big[\overline{Y}^2-\overline{Y}^2\alpha\theta g\Delta f\rho C_yC_x+\overline{Y}^2\frac{g(g+1)}{2}\alpha^2\theta^2\Delta f{C_x}^2\Big] \quad (50)$$

differentiating w.r.t k_1 and k_2 and equating it with zero we obtain the respective values

$$k_1 = \frac{1/\delta(GD + HB) - (CD - BE)}{(AD - B^2)} \tag{51}$$

$$k_2 = \frac{1/\delta(GD + HA) - (CD - AE)}{(AD - B^2)} \tag{52}$$

By setting the values of k_1 and k_2, we obtain the following:

$$MSE\,(t_g) = \frac{1}{(AD - B^2)^2}\left[\delta^2 J_1 - 2\delta J_2 + J_3 + \left(AD - B^2\right)^2 \overline{Y}^2\right]. \tag{53}$$

where

$$\begin{aligned} J_1 = {} & (CD - BE)^2 A + 2(CD - BE)(CB - AB)B \\ & - 2(CD - BE)\left(AD - B^2\right)C + (CB - AB)^2 D \\ & + 2(CB - AB)\left(AD - B^2\right)F + \left(AD - B^2\right)^2 F. \end{aligned} \tag{54}$$

$$\begin{aligned} J_2 = {} & (GD + HB)(CD - BE)A + 2(GD + HB)(CB - AB)B \\ & + 2(GB + HA)(CD - BE)B + 2(GB + HA)(CB - AB)D \\ & - 2(GD + HB)\big(AD - B^2\big)C + 2(GB + HA)\big(AD - B^2\big)D \\ & + \big(AD - B^2\big)^2 E - 2(CD - BE)\big(AD - B^2\big)G \\ & + 2(CB - AB)\big(AD - B^2\big)H - 2\big(AD - B^2\big)^2 I. \end{aligned} \tag{55}$$

$$\begin{aligned} J_3 = {} & (GD + HB)^2 A - 2(GD + HB)(GB + HA)B \\ & - 2(GD + HB)\left(AD - B^2\right)G + 2(GB + HA)\left(AD - B^2\right)H \\ & + (GB + HA)^2 D. \end{aligned} \tag{56}$$

Members of the proposed generalized class of estimators, denoted as t_g, are summarized in Tables 1, 2, and 3. These tables present various forms of the estimator obtained under specific choices of parameters and functional relationships, thereby illustrating the flexibility and wide applicability of the proposed family in different sampling situations.

Table 1 First class of estimators obtained from the generalized estimator T_g

k_1	k_2	δ	α	g	Estimator
0	0	0	0	0	$T_g^1 = W_t$
0	0	1	1	1	$T_g^2 = W_t \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
0	1	1	1	1	$T_g^5 = 2W_t + (\overline{Z} - Z_t)$
0	1	1	1	0	$T_g^6 = \left[W_t + W_t\left(\frac{\bar{Z}}{Z_t}\right) + (\bar{Z} - Z_t)\right] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
1	0	1	1	0	$T_g^7 = \left[W_t + W_t\left(\frac{\bar{Z}}{z_t}\right) + (\overline{Z} - z_t)\right]$
1	0	1	0	0	$T_g^9 = 2W_t$
1	0	1	1	0	$T_g^{10} = \left[W_t + W_t\left(\frac{\bar{Z}}{Z_t}\right)\right] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
1	0	0	0	0	$T_g^{12} = 2W_t$
0	0	1	1	0	$T_g^{13} = [W_t + (\bar{Z} - Z_t)] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
0	1	1	1	0	$T_g^{14} = W_t + (\bar{Z} - Z_t)$

Table 2 Second class of estimators obtained from the generalized estimator T_g

k_1	k_2	δ	α	g	**Estimator**
0	k_2	1	0	1	$T_g^{16} = \left[W_t + k_2(\bar{Z} - Z_t)\right]$
0	k_2	1	1	1	$T_g^{17} = \left[W_t + k_2(\bar{Z} - Z_t)\right] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
k_1	0	1	0	1	$T_g^{20} = W_t + k_1 W_t$
k_1	0	1	1	1	$T_g^{21} = \left[W_t + k_1 W_t\left(\frac{\bar{Z}}{Z_t}\right)\right] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
k_1	0	1	1	0	$T_g^{22} = \left[W_t + k_1 W_t \cdot \left(\frac{\bar{Z}}{Z_t}\right)\right]$
k_1	0	1	0	0	$T_g^{23} = W_t + k_1 W_t$
1	k_2	1	0	1	$T_g^{24} = 2W_t + k_2(\bar{Z} - Z_t)$
1	k_2	1	1	1	$T_g^{25} = \left[W_t + W_t\left(\frac{\bar{Z}}{Z_t}\right) + k_2(\bar{Z} - Z_t)\right] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
1	k_2	1	1	0	$T_g^{26} = \left[W_t + W_t\left(\frac{\bar{Z}}{Z_t}\right) + k_2(\bar{Z} - Z_t)\right]$

5 Empirical Study

Following the derivation of the proposed estimator and its theoretical properties, its performance is assessed through empirical study.

To evaluate the performance of the proposed class of memory-type estimators against existing estimators, two population datasets are considered.

Table 3 Third class of estimators obtained from the generalized estimator T_g

k_1	k_2	δ	α	g	Estimator
k_1	1	1	0	1	$T_g^{28} = \left[W_t + k_1 W_t + (\bar{Z} - Z_t)\right]$
k_1	1	1	1	1	$T_g^{29} = \left[W_t + k_1 W_t \left(\frac{\overline{Z}}{Z_t}\right) + (\overline{Z} - Z_t)\right] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
k_1	1	1	1	0	$T_g^{30} = \left[W_t + k_1 W_t \left(\frac{\bar{Z}}{Z_t}\right) + (\bar{Z} - Z_t)\right] \cdot \left(\frac{\bar{Z}}{Z_t}\right)$
k_1	1	1	0	0	$T_g^{31} = \left[W_t + k_1 W_t + (\bar{Z} - Z_t)\right]$
k_1	k_2	1	0	0	$T_g^{32} = \left[(1 + k_1)\, W_t + k_2(\bar{Z} - Z_t)\right]$

Table 4 Population parameters

Parameters	Population 1	Population 2
N	180	30
n	70	6
$\overline{Y}$	13.9951	0.686
$\overline{X}$	27.3981	4.6537
C_y	0.418	0.4803
C_x	0.4254	0.2295
ρ_{yx}	0.563	0.1794

Table 5 M.S.E. of estimators for different values of δ (Population 1)

δ	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_g
0.2	**0.0295372**	**0.1056178**	**0.0226738**	**0.227711**	**0.0570555**	**0.022670955**	**0.04357187**	**0.022644**
0.5	**0.0886117**	**0.3168534**	**0.0680214**	**0.0683134**	**0.1711664**	**0.067995702**	**0.130742689**	**0.0677575**
0.7	**0.1431420**	**0.5118402**	**0.1098808**	**0.1103525**	**0.2764995**	**0.109813632**	**0.211216773**	**0.1092020**
0.9	**0.2150150**	**0.7777312**	**0.1669617**	**0.1676785**	**0.4201356**	**0.166806703**	**0.320976144**	**0.1654259**

Population 1: The data is obtained from [19], and the measures are computed directly from the primary data.

Population 2: The data is obtained from [20]. The study variable Y represents the logarithm of leaf burn time (in seconds), while the auxiliary variable X corresponds to potassium percentage:

Y **= logarithm of leaf burn time (seconds)**
X **= potassium percentage**

Table 6 PRE for different values of δ (Population 1)

$\delta = 0.2$			$\delta = 0.5$		
Estimators	MSE	PRE	Estimators	MSE	PRE
T_0	0.2987629	100	T_0	0.2987629	100
T_1	0.0295372	1011.479	T_1	0.0886117	337.15967
T_2	0.1056178	282.87173	T_2	0.3168534	94.290577
T_3	0.0226738	1317.6562	T_3	0.0680214	439.21872
T_4	0.0227711	1312.0240	T_4	0.0683134	437.34135
T_5	0.0570555	523.63603	T_5	0.1711664	174.54534
T_6	0.022670955	1317.82229	T_6	0.067995702	439.38497
T_7	0.043577187	685.5947	T_7	0.130742689	228.51212
T_g	0.0226440	1319.3894	T_g	0.0677575	440.92959

Table 7 PRE for different values of δ (Population 1)

$\delta = 0.7$			$\delta = 0.9$		
Estimators	MSE	PRE	Estimators	MSE	PRE
T_0	0.2987629	100	T_0	0.2987629	100
T_1	0.1431420	208.71789	T_1	0.2150150	137.36135
T_2	0.5118402	58.370357	T_2	0.7777312	38.41468
T_3	0.1098808	271.89730	T_3	0.1669617	178.94096
T_4	0.1103525	270.73512	T_4	0.1676785	178.17610
T_5	0.2764995	108.05188	T_5	0.4201356	71.111065
T_6	0.109813632	272.06358	T_6	0.166806703	179.1072
T_7	0.211216773	141.44847	T_7	0.320976144	93.07947
T_g	0.1092020	273.58734	T_g	0.1654259	180.60229

Population 2: The MSEs and PREs using the data of population 2 are given below:

Table 8 M.S.E. of estimators for different values of δ (Population 2)

δ	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_g
0.2	**0.0016998**	**0.0022513**	**0.0015565**	**0.0015622**	**0.0019238**	**0.007776744**	**0.00178368**	**0.0015515**
0.5	**0.0050993**	**0.0067538**	**0.0046696**	**0.0046867**	**0.0057713**	**0.021837314**	**0.005320097**	**0.0046373**
0.7	**0.0082374**	**0.0109099**	**0.0075433**	**0.0075709**	**0.0093228**	**0.033665008**	**0.00854878**	**0.0074906**
0.9	**0.0125166**	**0.0165774**	**0.0114618**	**0.0115038**	**0.0141658**	**0.048699641**	**0.012898096**	**0.0114372**

Table 9 PRE for different values of δ (Population 2)

$\delta = 0.2$			$\delta = 0.5$		
Estimators	MSE	PRE	Estimators	MSE	PRE
T_0	0.0144748	100	T_0	0.0144748	100
T_1	0.0016998	851.56798	T_1	0.0050993	283.85599
T_2	0.0022513	642.96639	T_2	0.0067538	214.32213
T_3	0.0015565	929.92918	T_3	0.0046696	309.97639
T_4	0.0015622	926.53841	T_4	0.0046867	308.84614
T_5	0.0019238	752.42427	T_5	0.0057713	250.80809
T_6	0.007776744	186.12931	T_6	0.021837314	66.2847
T_7	0.00178368	811.51327	T_7	0.005320097	272.0777
T_g	0.0015515	932.98101	T_g	0.0046373	312.13623

Table 10 PRE for different values of δ (Population 2)

$\delta = 0.7$			$\delta = 0.9$		
Estimators	MSE	PRE	Estimators	MSE	PRE
T_0	0.0144748	100	T_0	0.0144748	100
T_1	0.0082374	175.72038	T_1	0.0125166	115.64503
T_2	0.0109099	132.67560	T_2	0.0165774	87.316423
T_3	0.0075433	191.89015	T_3	0.0114618	126.28668
T_4	0.0075709	191.19047	T_4	0.0115038	125.82620
T_5	0.0093228	155.26215	T_5	0.0141658	102.18107
T_6	0.033665008	42.99657	T_6	0.048699641	29.7226
T_7	0.00854878	169.3200	T_7	0.012898096	112.2243
T_g	0.0074906	193.23809	T_g	0.0114372	126.55857

6 Simulation Study

The mean square error and Percent relative efficiency are computed by using the formulae:

$$MSE(t) = \frac{1}{10000} \sum_{i=1}^{10000} (t_i - \mu_Y)^2 \tag{57}$$

where, $t = T_i, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_g$ and μ_Y is the mean of t from 10000 samples.

$$PRE(t) = \frac{MSE(\bar{y})}{MSE(t_i)} \times 100 \tag{58}$$

where, $\bar{y}$ is the mean per unit estimator.

Table 11 MSE of proposed estimator T_g at different values of ρ, δ, and n

ρ	n	0.05	0.1	0.25	0.5	0.75	1
0.250000	10.000000	0.000566	0.000972	0.010244	0.053454	0.127182	0.248443
0.500000	10.000000	0.000208	0.000881	0.007915	0.036649	0.088005	0.157940
0.750000	10.000000	0.000120	0.000799	0.007192	0.032157	0.073549	0.131751
0.950000	10.000000	0.000086	0.000739	0.006952	0.030674	0.071601	0.129380
0.250000	20.000000	0.000129	0.000746	0.006372	0.029288	0.068227	0.121434
0.500000	20.000000	0.000114	0.000609	0.004629	0.019429	0.046086	0.081583
0.750000	20.000000	0.000101	0.000537	0.003954	0.016787	0.039409	0.068368
0.950000	20.000000	0.000095	0.000512	0.003890	0.016009	0.037857	0.065506
0.250000	30.000000	0.000108	0.000589	0.004582	0.019865	0.045226	0.078442
0.500000	30.000000	0.000089	0.000446	0.003131	0.012969	0.030574	0.053201
0.750000	30.000000	0.000084	0.000404	0.002752	0.011455	0.025578	0.046287
0.950000	30.000000	0.000077	0.000374	0.002638	0.011107	0.024931	0.043859
0.250000	50.000000	0.000086	0.000410	0.002935	0.012113	0.028403	0.051046
0.500000	50.000000	0.000063	0.000283	0.001950	0.008100	0.018053	0.032120
0.750000	50.000000	0.000058	0.000257	0.001721	0.007088	0.015918	0.028223
0.950000	50.000000	0.000056	0.000240	0.001588	0.006428	0.015364	0.026028
0.250000	200.000000	0.000032	0.000132	0.000866	0.003449	0.008029	0.013409
0.500000	200.000000	0.000017	0.000071	0.000452	0.001823	0.004133	0.007477
0.750000	200.000000	0.000017	0.000069	0.000444	0.001779	0.004078	0.007147
0.950000	200.000000	0.000015	0.000062	0.000403	0.001583	0.003586	0.006514
0.250000	500.000000	0.000017	0.000069	0.000444	0.001690	0.003889	0.007002
0.500000	500.000000	0.000006	0.000026	0.000156	0.000634	0.001379	0.002491
0.750000	500.000000	0.000008	0.000030	0.000187	0.000742	0.001659	0.002972
0.950000	500.000000	0.000006	0.000022	0.000145	0.000574	0.001267	0.002234

By following the steps listed below, we computed the mean square error and relative efficiency of the proposed estimator.

1. Using the bi-variate normal distribution with parameters $(Y, X) \sim N_2$ $(2, 10, 1, 2, \rho)$ where $\rho = \{0.25, 0.50, 0.75, 0.95\}$ and $\delta = \{0.05, 0.1, 0.25, 0.50, 0.75, 1\}$, generate a population of size 5000.
2. Choose the values for δ.
3. We selected 10000 samples of six different sizes such as $n = 10, 20, 30, 50, 200$ and 500.

7 Discussion

The empirical and simulation results mentioned above are interpreted and compared in detail in the following discussion. The comparative performance of the proposed estimator T_g with the existing estimators T_0–T_5 was examined for varying values of the smoothing parameter δ using two distinct real

Table 12 Percentage Relative Efficiency (PRE) of proposed estimator T_g at different values of ρ and n

ρ	n	0.05	0.1	0.25	0.5	0.75	1
0.250000	10.000000	17653.457579	10286.047515	976.178302	187.075043	78.627486	40.250756
0.500000	10.000000	47977.340470	11344.634098	1263.454572	272.855256	113.629969	63.315058
0.750000	10.000000	83073.968421	12517.921701	1390.512765	310.976869	135.963974	75.900846
0.950000	10.000000	116691.793518	13532.024128	1438.359162	326.003996	139.663639	77.291410
0.250000	20.000000	38677.615051	6704.566578	784.730567	170.719332	73.284830	41.174508
0.500000	20.000000	43729.724609	8205.102104	1080.053980	257.353234	108.492629	61.287405
0.750000	20.000000	49678.242467	9311.812811	1264.478192	297.841302	126.873536	73.134005
0.950000	20.000000	52770.924582	9759.024340	1285.226791	312.331113	132.074233	76.329080
0.250000	30.000000	30891.014715	5662.191699	727.518414	167.798645	73.703724	42.494467
0.500000	30.000000	37322.096926	7473.661171	1064.528199	257.024094	109.025299	62.655178
0.750000	30.000000	39854.083806	8249.311174	1211.415310	290.981982	130.318544	72.014037
0.950000	30.000000	43014.947854	8902.814774	1263.696322	300.122319	133.704017	76.001834
0.250000	50.000000	23312.551298	4880.907848	681.531631	165.114329	70.415474	39.180707
0.500000	50.000000	31961.979428	7056.759846	1025.539774	246.926033	110.785230	62.267079
0.750000	50.000000	34647.717845	7785.474573	1162.448503	282.155597	125.645558	70.864494
0.950000	50.000000	35697.616921	8350.634789	1259.738318	311.121216	130.173785	76.841440
0.250000	200.000000	15769.040404	3780.212002	577.224696	144.974091	62.274231	37.289271
0.500000	200.000000	28733.143402	7068.408149	1107.379166	274.317700	120.991306	66.871686
0.750000	200.000000	29109.073921	7245.510137	1125.564697	281.049967	122.607721	69.961716
0.950000	200.000000	32416.263782	8004.076359	1239.517503	315.952309	139.418503	76.753147
0.250000	500.000000	11991.662237	2896.572439	450.177486	118.342033	51.423535	28.564877
0.500000	500.000000	31442.149065	7655.668586	1278.314089	315.492492	145.069644	80.294441
0.750000	500.000000	26208.675230	6716.289198	1068.315775	269.533674	120.585666	67.289349
0.950000	500.000000	35999.734091	8972.162939	1376.727979	348.205204	157.819259	89.514043

populations. The parameter δ was varied between 0 and 1 to investigate how the relative weighting of past and recent observations influences estimator performance. Smaller values of δ emphasize historical information, while larger values place greater weight on more recent data. The results indicate that the efficiency of all estimators is sensitive to changes in δ; however, the proposed estimator T_g consistently demonstrates superior performance across all cases, characterized by minimum Mean Square Error (MSE) and maximum Percent Relative Efficiency (PRE).

The empirical evaluation of Populations 1 and 2 demonstrates the effectiveness of incorporating the EWMA framework using auxiliary information under SRS. The baseline characteristics reveal that Population 1 exhibits a moderate positive correlation between the study and auxiliary variables ($\rho_{yx} = 0.563$), while Population 2 shows a relatively weak correlation ($\rho_{yx} = 0.1794$). These differences provide an opportunity to examine the robustness of the proposed estimators under varying correlation structures.

In Population 1, the proposed memory-type estimators achieve substantial reductions in MSE (as can be seen in Table 5 compared to existing estimators for all tested values of δ. The smallest MSE occurs at $\delta = 0.2$,

Table 13 Mean Square Error (MSE) of ratio-type estimators T_1, T_4, and T_7 at different values of ρ, δ, and n

ρ	n	δ	MSE_{T_1}	MSE_{T_4}	MSE_{T_7}
0.250000	10.000000	0.050000	0.000236	0.000234	4.000000
0.500000	10.000000	0.050000	0.000195	0.000216	4.000000
0.750000	10.000000	0.050000	0.000138	0.000182	4.000000
0.950000	10.000000	0.050000	0.000103	0.000170	4.000000
0.250000	20.000000	0.050000	0.000117	0.000115	4.000000
0.500000	20.000000	0.050000	0.000098	0.000109	4.000000
0.750000	20.000000	0.050000	0.000070	0.000092	4.000000
0.950000	20.000000	0.050000	0.000052	0.000085	4.000000
0.250000	30.000000	0.050000	0.000079	0.000078	4.000000
0.500000	30.000000	0.050000	0.000065	0.000072	4.000000
0.750000	30.000000	0.050000	0.000047	0.000063	4.000000
0.950000	30.000000	0.050000	0.000034	0.000057	4.000000
0.250000	50.000000	0.050000	0.000048	0.000047	4.000000
0.500000	50.000000	0.050000	0.000039	0.000043	4.000000
0.750000	50.000000	0.050000	0.000028	0.000037	4.000000
0.950000	50.000000	0.050000	0.000021	0.000035	4.000000
0.250000	200.000000	0.050000	0.000011	0.000011	4.000000
0.500000	200.000000	0.050000	0.000010	0.000011	4.000000
0.750000	200.000000	0.050000	0.000007	0.000009	4.000000
0.950000	200.000000	0.050000	0.000005	0.000009	4.000000
0.250000	500.000000	0.050000	0.000004	0.000004	4.000000
0.500000	500.000000	0.050000	0.000004	0.000004	4.000000
0.750000	500.000000	0.050000	0.000003	0.000004	4.000000
0.950000	500.000000	0.050000	0.000002	0.000004	4.000000

resulting in PRE values above 1300%, which reflects a more than thirteen fold improvement in efficiency relative to the baseline. The efficiency advantage diminishes as δ increases toward 0.9, consistent with the property that higher δ values emphasize recent observations. The superior performance at smaller values of δ is attributed to the longer memory of the EWMA statistic, which effectively smooths random sampling fluctuations. In contrast, larger values of δ place greater emphasis on recent observations, increasing responsiveness but also estimator variability.

For **Population 1**, T_g attained the lowest MSE and the highest PRE for every considered value of δ. As given in Table 6 The estimator reached its peak efficiency at $\delta = 0.2$, achieving a PRE of 1319.3894 substantially higher than those of competing estimators. This observation implies that

Table 14 Mean Square Error (MSE) of product-type estimators T_2, T_3, T_5, and T_6 at different values of ρ, δ, and n

ρ	n	δ	MSE_{T_2}	MSE_{T_3}	MSE_{T_5}	MSE_{T_6}
0.250000	10.000000	0.050000	0.000347	0.000266	0.000304	64.000000
0.500000	10.000000	0.050000	0.000406	0.000219	0.000265	64.000000
0.750000	10.000000	0.050000	0.000434	0.000125	0.000165	64.000000
0.950000	10.000000	0.050000	0.000494	0.000028	0.000068	64.000000
0.250000	20.000000	0.050000	0.000167	0.000122	0.000142	64.000000
0.500000	20.000000	0.050000	0.000205	0.000102	0.000123	64.000000
0.750000	20.000000	0.050000	0.000219	0.000059	0.000081	64.000000
0.950000	20.000000	0.050000	0.000250	0.000013	0.000033	64.000000
0.250000	30.000000	0.050000	0.000112	0.000081	0.000093	64.000000
0.500000	30.000000	0.050000	0.000135	0.000066	0.000080	64.000000
0.750000	30.000000	0.050000	0.000148	0.000038	0.000051	64.000000
0.950000	30.000000	0.050000	0.000165	0.000008	0.000022	64.000000
0.250000	50.000000	0.050000	0.000067	0.000048	0.000056	64.000000
0.500000	50.000000	0.050000	0.000082	0.000039	0.000047	64.000000
0.750000	50.000000	0.050000	0.000088	0.000023	0.000031	64.000000
0.950000	50.000000	0.050000	0.000102	0.000005	0.000014	64.000000
0.250000	200.000000	0.050000	0.000016	0.000011	0.000013	64.000000
0.500000	200.000000	0.050000	0.000021	0.000010	0.000012	64.000000
0.750000	200.000000	0.050000	0.000021	0.000006	0.000008	64.000000
0.950000	200.000000	0.050000	0.000026	0.000001	0.000003	64.000000
0.250000	500.000000	0.050000	0.000006	0.000004	0.000005	64.000000
0.500000	500.000000	0.050000	0.000009	0.000004	0.000004	64.000000
0.750000	500.000000	0.050000	0.000008	0.000002	0.000003	64.000000
0.950000	500.000000	0.050000	0.000010	0.000000	0.000001	64.000000

placing slightly greater emphasis on past data leads to enhanced stability and precision in estimation. As δ increased toward 1 (Table 7) the MSE values of all estimators rose gradually, while their PREs declined, reflecting a shift toward recent information. Nevertheless, T_g consistently maintained its efficiency advantage, demonstrating its robustness against changes in temporal weighting and its adaptability to differing data dependencies.

In Population 2, despite the lower correlation between Y and X, the proposed estimators maintain a significant performance edge, with PRE values exceeding 900% at $\delta = 0.2$. This finding is particularly notable because it suggests that the EWMA-based approach can still extract efficiency gains even when the auxiliary variable is only weakly related to the study variable—likely due to the smoothing effect of the EWMA, which stabilizes the estimator by integrating information from both current and past samples. The gradual decline in PRE as δ increases mirrors the behavior in Population 1, highlighting the importance of careful selection of the smoothing constant to balance stability and adaptability.

A comparable pattern emerged for **Population 2**, where T_g again produced the smallest MSE (given in Table 8) and the highest PRE across all values of δ. The maximum efficiency was recorded at $\delta = 0.2$ with a PRE of 932.9810 (given in Table 9) confirming that moderate weighting toward past observations yields optimal precision. Although a slight reduction in efficiency occurred as δ approached 1, T_g continued to outperform the other estimators, maintaining superior accuracy and consistency even when recent data exerted stronger influence. The gradual decline in performance across all estimators at higher δ values suggests that overemphasizing recent information can increase estimator variability, yet T_g remained the most stable and efficient alternative under these conditions. The results further indicate that the strength of the correlation between the study and auxiliary variables influences the magnitude of the efficiency gain. The proposed estimator performs consistently across different correlation structures, with the efficiency gains being better for positive correlation.

The Tables 11–13 depict the empirical evidence using both population 1 and population 2 that the proposed estimator T_g provides substantial efficiency gains over the existing estimators T_0–T_5 across all values of δ. Its ability to maintain low MSE and high PRE under varying temporal weights indicates that T_g effectively balances the contribution of both historical and recent sample information. This adaptability enhances its reliability in diverse sampling environments, establishing T_g as a robust and efficient alternative, particularly in situations where the smoothing parameter δ is used to control the dynamic influence of past and present observations on estimation precision. Overall, these results confirm that the proposed EWMA type estimators offer consistent and substantial improvements over conventional estimators. The simulation results show that larger sample sizes reduce the mean squared error, while the proposed estimator maintains its efficiency across all sample sizes considered. The empirical evidence supports the theoretical premise that smaller δ values optimize the trade-off between variance reduction and responsiveness, producing estimators that are both precise and robust in diverse sampling environments. The adaptability of the method makes it suitable for applications where population characteristics evolve over time, reinforcing its value as a practical and efficient alternative in survey sampling. In practical setting, the proposed estimator can be used in longitudinal surveys, environmental applications, and other surveys where information is collected sequentially and where past information can be incorporated under EWMA framework.

8 Conclusion

The following section presents the key conclusions drawn from the findings of this study. The above study demonstrates that integrating the EWMA framework significantly enhances the precision of population mean estimation under simple random sampling. By leveraging auxiliary information and optimally selecting the smoothing constant δ, the proposed estimators achieve consistent reductions in mean squared error across populations with varying correlation structures. The empirical results confirm that smaller δ values strike an effective balance between incorporating historical data and responding to recent observations, leading to substantial efficiency gains over existing estimators. In particular, the results indicate that appropriate selection of the EWMA smoothing constant enables an effective balance between past and current sample information, thereby improving estimation precision. These findings position the proposed estimators as a practical and effective tool for survey practitioners, particularly in applications where population characteristics evolve over time or where data are collected sequentially.

Although the proposed methodology demonstrates improved estimation efficiency, it is subject to certain limitations. Its theoretical properties are established using first-order Taylor series approximations under the assumption of simple random sampling without replacement. Furthermore, the estimator depends on the availability of reliable auxiliary information, and its performance varies according to the extent of association between the auxiliary and study variables.

Future research may explore the extension of the proposed estimator to higher-order approximations, complex sampling designs, and non-normal population distributions. Furthermore, the methodology may be expanded by incorporating multiple auxiliary variables and investigating data-driven approaches for selecting the EWMA smoothing parameter.

Authors' Declaration

Originality: The authors declare that this manuscript is an original work and has not been published previously, nor is it under consideration for publication elsewhere.

Authorship Contribution: All authors have substantially contributed to the conception, design, methodology, analysis, interpretation of results, drafting, and preparing the final version of the manuscript. All authors have read and approved the final version of the manuscript and agree to be accountable for its contents.

Ethical Approval: This study does not involve human participants or animals; hence, ethical approval is not required.

Funding: No external funding was received for conducting this study.

Conflict of Interest: The authors declare that they have no conflict of interest, financial or otherwise, related to the publication of this work.

Data Availability: All data used in this study are included within the manuscript. Additional information can be provided by the corresponding author upon reasonable request.

Acknowledgment: The authors gratefully acknowledge all individuals and institutions who supported this work.

References

[1] W. G. Cochran, "The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce," *J. Agric. Sci.*, vol. 30, no. 2, pp. 262–275, 1940.

[2] D. S. Robson, "Applications of Multivariate Polykays to the Theory of Unbiased Ratio-Type Estimation," *J. Am. Stat. Assoc.*, vol. 52, no. 280, pp. 511–522, Dec. 1957, doi: 10.1080/01621459.1957.10501407.

[3] C. Kadilar and H. Cingi, "Ratio estimators in simple random sampling," *Appl. Math. Comput.*, vol. 151, no. 3, pp. 893–902, Apr. 2004, doi: 10.1016/S0096-3003(03)00803-8.

[4] M. Noor-ul-Amin, "Memory type ratio and product estimators for population mean for time-based surveys," *J. Stat. Comput. Simul.*, vol. 90, no. 17, pp. 3080–3092, Nov. 2020, doi: 10.1080/00949655.2020.1795660.

[5] S. A. Ravindrabahadur, R. Singh, and S. N. Tiwari, "Logarithmic Ratio-Type Estimator for Finite Population Mean under Neutrosophic Framework," *Neutrosophic Optim. Intell. Syst.*, vol. 7, pp. 11–24, Aug. 2025, doi: 10.61356/j.nois.2025.7571.

[6] R. Singh, A. Kumari, F. Smarandache, and S. N. Tiwari, "Construction Of Almost Unbiased Estimator For Population Mean Using Neutrosophic Information", *Journal of Statistics Applications*, 2025.

[7] S. K. Yadav, R. Singh, and S. N. Tiwari, "Estimation of population mean utilizing two neutrosophic auxiliary variables with imprecise information," *Uncertain. Discourse Appl.*, vol. 2, no. 1, pp. 17–31, Mar. 2025, doi: 10.48313/uda.v2i1.51.

[8] Kumari, M., Sharma, P., Singh, P., and Ozel, G. Enhanced Mean Estimation Using Memory Type Estimators with Dual Auxiliary Variables: Accepted – January 2025. REVSTAT-Statistical Journal. https://revstat.ine.pt/index.php/REVSTAT/article/view/892

[9] P. Sharma, M. Kumari, and P. Singh, "Advancing Stratified Sampling: Optimized Mean Estimation via Linear Cost Structures," *J. Stat. Theory Pract.*, vol. 19, no. 3, p. 55, Sept. 2025, doi: 10.1007/s42519-025-00470-2.

[10] P. Singh, P. Sharma, and P. Maurya, "Enhancing Accuracy in Population Mean Estimation with Advanced Memory Type Exponential Estimators," *Journal of Reliability and Statistical Studies*, vol. 17, no. 2, pp. 417–434, 2024. doi: 10.13052/jrss0974-8024.1728.

[11] P. Sharma, A. Lata, S. K. Yadav, and M. Noor-ul-Amin, "Family of Estimators for Estimating Population Median using Auxiliary Information in Survey Sampling," *Journal of Reliability and Statistical Studies*, vol. 18, no. 2, pp. 343–370, 2025. doi: 10.13052/jrss0974-8024.1824.

[12] R. Singh and P. Sharma, "A Class of Exponential Ratio Estimators of Finite Population Mean Using Two Auxiliary Variables," *Pak. J. Stat. Oper. Res.*, vol. 11, no. 2, p. 221, Aug. 2015, doi: 10.18187/pjsor.v11i2.759.

[13] M. Noor-ul-Amin, "Memory type estimators of population mean using exponentially weighted moving averages for time scaled surveys," *Commun. Stat. - Theory Methods*, vol. 50, no. 12, pp. 2747–2758, June 2021, doi: 10.1080/03610926.2019.1670850.

[14] D. J. Watson, "The estimation of leaf area in field crops," *J. Agric. Sci.*, vol. 27, no. 3, pp. 474–483, July 1937, doi: 10.1017/S002185960005173X.

[15] P. Singh, P. Sharma, and P. Maurya, "Enhancing Accuracy in Population Mean Estimation with Advanced Memory Type Exponential Estimators," *J. Reliab. Stat. Stud.*, pp. 417–434, Jan. 2025, doi: 10.13052/jrss0974-8024.1728.

[16] S. Bahl and R. K. Tuteja, "Ratio and Product Type Exponential Estimators," *J. Inf. Optim. Sci.*, vol. 12, no. 1, pp. 159–164, Jan. 1991, doi: 10.1080/02522667.1991.10699058.

[17] I. Aslam, M. N. Amin, A. Mahmood, and P. Sharma, "New memory-based ratio estimator in survey sampling," *Nat. Appl. Sci. Int. J. NASIJ*, vol. 5, no. 1, pp. 168–181, June 2024, doi: 10.47264/idea.nasij/5.1.11.

[18] P. Sharma, P. Singh, M. Kumari, and R. Singh, "Estimation Procedures for Population Mean using EWMA for Time Scaled Survey," *Sankhya B*, vol. 87, no. 1, pp. 103–128, May 2025, doi: 10.1007/s13571-024-00347-7.

[19] S. Y. Feng and X. Q. Shi, *The Sampling Survey—Theory, Method and Practice*. Shanghai, China: Shanghai Scientific and Technical Publishers, 1996.

[20] R. G. D. Steel and J. H. Torrie, *Principles and Procedures of Statistics*. New York: McGraw-Hill, 1960.

[21] Zaman, T., Itikhar, S., Sozen, C., and Sharma, P. A new logarithmic type estimators for analysis of number of aftershocks using poisson distribution. *Journal of Science and Arts*, 24(4), 833-842.2024

[22] Zaman, T. and Bulut, H. A simulation study: Robust ratio double sampling estimator of finite population mean in the presence of outliers. *Scientia Iranica*, 31(15), 1330-1341.2024

[23] P. Sharma, P. Singh, M. Kumari, Abhilasha, and A.K. Misra. Harnessing qualitative data for precision in population mean estimation in time-scaled surveys. *Quality & Quantity*, 60(2), 6599–6618, 2026.

Biographies

Prayas Sharma is currently working as Assistant Professor in the Department of Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow.

Dr. Sharma has good knowledge of Statistics, Artificial Intelligence and Machine Learning, Business Analytics & Research Methodology along with strong computational & programming skills. He has more than 11 years of academic experience, both in the domain of teaching and research. His research interest includes Survey Sampling, Estimation Procedures using Auxiliary Information and Measurement Errors, Predictive Modelling, Business Analytics and Operations Research. Dr. Sharma has published more than 85 research papers in reputed National & International journals along with one book and two chapters in book internationally published. He has more than 1050 citations with H-Index 20 & I index of 30. Dr. Sharma has a keen interest in reading, writing and publishing, he is serving 10 reputed journals as editor.

Rajesh Singh is a Professor in the Department of Statistics, Banaras Hindu University (BHU), Varanasi, India. With more than 30 years of academic and research experience, he has published more than 200 research papers in reputed national and international journals. His research interests primarily encompass survey sampling, finite population estimation, auxiliary information, statistical modelling, and the development of efficient estimators for population parameters. Prof. Singh has made significant contributions to sampling theory and estimation methodology and has supervised more than 20 research scholars in these areas. His extensive research experience and contributions have established him as an active researcher in the field of statistics. He continues to be actively engaged in teaching, research, research supervision, and various academic activities at Banaras Hindu University.

Yashveer Singh is a Research Scholar in the Department of Statistics, Banaras Hindu University (BHU), Varanasi, India. He holds a Master's degree in Statistics and is currently pursuing his research in the area of sampling theory and statistical estimation. He has published three research papers in reputed journals and has been recognized by Banaras Hindu University for his contribution to quality research publications. His research interests focus on developing efficient statistical methodologies and estimation techniques in survey sampling.